Public Finance Review

Editor
J. Ronnie Davis, University of New Orleans

Associate Editors
James R. Alm, Georgia State University
John E. Filer, University of South Alabama

Editorial Board
Henry Aaron, The Brookings Institution
J. Gregory Ballentine, KPMG Peat Marwick
Richard Bird, University of Toronto
H. Geoffrey Brennan, Australian National University
Benjamin Bridges, Social Security Administration
Edgar Browning, Texas A&M University
Richard J. Cebula, Armstrong Atlantic State University
Martin David, University of Wisconsin
David G. Davies, Duke University
Martin Feldstein, Harvard University
John E. Filer, University of South Alabama
Bruce Hamilton, Johns Hopkins University
Vernon Henderson, Brown University
Harold M. Hochman, Lafayette College
Randall G. Holcombe, Florida State University
Robert Inman, University of Pennsylvania
Henry M. Levin, Columbia University Teachers College
Charles E. McLure, Jr., Hoover Institution
Peter Mieszkowski, Rice University
Richard Musgrave, University of California, Santa Cruz
Dick Netzer, New York University
William Oakland, Tulane University
Wallace Oates, University of Maryland
Mark V. Pauly, University of Pennsylvania
Harvey Rosen, Princeton University
Daniel Rubinfeld, University of California, Berkeley
John Shoven, Stanford University
Nicolaus Tideman, Virginia Polytechnic Institute
Burton Weisbrod, Northwestern University

For Sage Publications: Scott Springer, Matthew H. Adams,
Joyce Kuhn, and Paul Doebler
CONTENTS

Imperfect Competition and Indirect Tax Structure in a Deregulated Telecommunications Sector
FITZROY A. LEE 419

Shadow Prices of Missouri Public Conservation Land
ROLF FÄRE
SHAWNA GROSSKOPF
WILLIAM L. WEBER 444

Specific Inputs, Value-Added, and Production Linkages in Tax-Incidence Theory
KUL B. BHATIA 461

Index 487
This article uses an applied general equilibrium model to evaluate the relative efficiency of alternative indirect tax structures on the telecommunications sector. The article especially investigates the impact of imperfect competition in the sector’s product market on the efficiency costs of the indirect tax structures. The results show that for a shift from differential capital taxation to uniform capital taxation, the marginal efficiency gains under imperfect competition are 1½ to 2 times larger than the marginal efficiency gains under perfect competition. The results also show that although a differential capital tax has an efficiency advantage over a differential commodity tax under perfect competition under imperfect competition, a differential commodity tax is more efficient.

IMPERFECT COMPETITION AND INDIRECT TAX STRUCTURE IN A DEREGULATED TELECOMMUNICATIONS SECTOR

FITZROY A. LEE
Tulane University

1. INTRODUCTION

The passage of the Telecommunications Act of 1996 deregulating the telecommunications industry has sparked telecommunications tax reform debates in many states. One tax reform issue that often emerges in the debates is the differentially higher average tax rates on telecommunications property in many states. Traditionally, differentially higher taxation of the sector is, in part, a quid pro quo for monopoly privileges, the power of eminent domain, and the use of public rights-of-way. But as the sector becomes more competitive, industry leaders and policy makers have begun to reexamine the efficacy of this arrangement.

Many industry analysts cite differential capital taxation as a major source of inefficiency in the sector and predict large efficiency gains for a shift to uniform capital taxation of the sector (Case 1992;
Missing from these analyses are actual measures of the efficiency costs of differential capital taxation of the sector. Predictions of efficiency gains are largely based on efficiency costs calculated for capital taxes elsewhere in the literature. But the efficiency costs of differential capital taxation of the telecommunications sector may depart significantly from calculations of the efficiency costs of capital taxes in the literature. The reason is that estimates of the efficiency costs of capital taxes assume a perfectly competitive product market, whereas the telecommunications product market is likely to be imperfectly competitive. Even though the Telecommunications Act of 1996 removes the regulatory barriers to entry in the telecommunications sector, imperfect competition nevertheless may arise because the huge setup costs that still characterize the sector limit the entry of firms into the sector. Even so, imperfect competition in the telecommunications product market is likely to increase the efficiency costs of differential capital taxation, strengthening the case for tax reform.

However, the property tax on the telecommunications sector currently funds school districts. Given the current emphasis on improving school performance, any tax reform that eliminates the capital tax differential must propose feasible alternatives to recoup lost revenues. But any alternative tax structure besides a lump-sum tax is likely to incur its own efficiency costs. The question then is whether the alternative tax structure has an efficiency advantage over a differential capital tax. But estimates of efficiency costs of capital taxes in the literature often are not based on revenue neutral tax reforms, or if they are, the alternative tax structure is a lump-sum tax, which is generally not feasible.

This article aims to shift the debates onto more empirical grounds by quantifying the efficiency gains of common tax reform proposals for the telecommunications sector. The article uses a numerical general equilibrium model, calibrated to 1992 data for Georgia. For each tax reform, efficiency gains are calculated for the alternative market structure assumptions of perfect competition and imperfect competition. The contribution of the article is threefold. First, the article calculates, for the first time, the order of magnitude efficiency gains of shifting from differential taxation to uniform taxation of the telecommunica-
tions sector. The results show that for the perfect competition case, the marginal efficiency gains for a shift to uniform capital taxation is of the same orders of magnitudes as the efficiency costs of capital taxes calculated elsewhere, ranging from 7¢ to 21¢ for each additional dollar of revenue.

Second, the article shows that taking into account imperfect competition has a substantial impact on the calculated marginal efficiency gains. More specific, the marginal efficiency gains calculated for the imperfect competition case are from 1½ to more than 2 times that for the perfect competition case, ranging from 19¢ to 35¢ for each additional dollar of revenue. Third, the article shows that for differential taxation of the sector, allowing for imperfect competition may change the relative efficiency of alternative tax instruments. For example, the results show that under perfect competition, a differential capital tax has an efficiency advantage over a differential commodity tax. But under imperfect competition, a differential commodity tax may be more efficient.

The rest of the article is organized as follows. Section 2 shows how, under imperfect competition, a capital tax generates indirect efficiency losses in addition to its direct efficiency costs. Section 3 specifies the general equilibrium tax model with imperfect competition. Section 4 presents the benchmark data and specifies the exogenous parameters of the model. Section 5 describes the results, and Section 6 summarizes them and discusses their implications and limitations.

2. WHY IMPERFECT COMPETITION MATTERS

Under imperfect competition, in addition to its direct efficiency costs, a tax generates indirect efficiency costs that are not present under perfect competition. The following heuristic analysis, due to Willig (1983), illustrates this point for a capital tax. Let firm $i$’s profit be given by

$$\Pi^i = P^i Q^i - C(Q^i, P, r + t_1),$$

(2.1)

where $P$ and $Q$ are the price and quantity in the firm’s product market, and $C(.)$ is the firm’s cost function. $P$ is the vector of prices of the
economy’s produced goods, \( r \) is the rental rate of capital, and \( t_i \) is the tax on the firm’s capital. The model assumes that only firm \( i \) is taxed and that wage is fixed at unity. For the analysis, I use a social welfare measure, \( W \), which comprises consumers’ surplus (\( CS \)), the firm’s profit (\( \Pi \)), and the total tax proceeds (\( T \)):

\[
W = \Pi + CS + T.
\]

The effect of a change in the tax on efficiency is found by taking the derivative of each component of social welfare with respect to \( t_i \):

\[
\frac{d\Pi}{dt_i} = P^i \frac{dQ^i}{dt_i} + Q^i \frac{dP^i}{dt_i} - C^i Q^i \frac{dQ^i}{dt_i} - C^i \left(1 + \frac{dr}{dt_i}\right) - \sum_j C^i \frac{dP^j}{dt_i},
\]

\[
\frac{dCS}{dt_i} = D^i \frac{dP^i}{dt_i},
\]

\[
\frac{dT}{dt_i} = t_i \frac{dK^i}{dt_i} + K^i,
\]

where \( D^i \) is the total consumer demand for the firm’s output, and \( K^i \) is the amount of capital employed by the firm.

Summing these first-order effects and substituting \( K^i = C^i \) and \( Q^i = C^i \) (Shephard’s lemma), where \( Q^i \) is the quantity of good \( j \) employed by firm \( i \),

\[
\frac{dW}{dt_i} = t_i \frac{dK^i}{dt_i} + \left(P^i - C^i\right) \frac{dQ^i}{dt_i} + \left(Q^i - D^i\right) \frac{dP^i}{dt_i} - \left[K^i \frac{dr}{dt_i} + \sum_j Q^{ij} \frac{dP^j}{dt_i}\right].
\]

Shephard’s lemma applies because the firm is a price taker in its input market and minimizes cost.

Each term in the expression relates to a component of efficiency loss that arises from the imposition of the capital tax. The first term is the direct inefficiency effect of the tax. The tax, \( t_i \), is a wedge between the private and the social costs of capital, and the higher the tax, the less capital is used. The second term is the output repression effect of the tax. It arises because if price is above marginal cost, then any tax-induced reduction of output lowers efficiency. The third term is the
change in the firm’s profits that are obtained by applying the price change induced by the tax to that portion of the firm’s output that are intermediate inputs to other firms. In a total efficiency analysis, this effect will somewhat offset the pass-through effects on the profits of intermediate purchasers of the taxed firm’s output and the welfare of customers of the intermediate purchasers. The fourth term (in square brackets) represents feedback effects on the firm’s profits through the tax-induced changes in the input prices that the firm faces. Again, in a total efficiency analysis, this effect will somewhat offset the effect on the profits of suppliers of inputs to the taxed firm.

When the capital tax is imposed on a firm whose product market is perfectly competitive, only the direct inefficiency effect applies. There is no output repression effect because price is equal to marginal costs and tax-induced changes in output have no efficiency effect. Any pass-through effects on intermediate purchasers and their customers is exactly offset by the change in the taxed firm’s profits. Similarly, any feedback effects on the taxed firm’s profits is exactly offset by the effects on the profits of the suppliers of its inputs. However, when the product market of the taxed firm is imperfectly competitive, these indirect effects are no longer zero and combine with the direct effects to increase the efficiency costs of the tax over the perfectly competitive case.

3. AN APPLIED GENERAL EQUILIBRIUM TAX MODEL WITH IMPERFECT COMPETITION

A numerical general equilibrium model, calibrated to 1992 data for the state of Georgia, is employed to explore the implications of imperfect competition for telecommunications tax reform. The model is a static, tax-based general equilibrium model in the vein of Shoven and Whalley (1992). Like the Shoven and Whalley model, the economy is treated as closed, and labor and capital are assumed to be in aggregate fixed supply although perfectly mobile between sectors. The innovation on the Shoven and Whalley model is the modeling of the telecommunications sector as imperfectly competitive. Imperfect competition in the telecommunications sector is modeled as arising from setup
costs that limit the entry of firms into the sector. Specifically, the telecommunications sector is assumed to be monopolistically competitive, with product differentiation by firm. Elasticity of scale estimates from engineering studies is used to calibrate the imperfectly competitive telecommunications sector. The calibrated model is then used to evaluate the relative efficiency of alternative telecommunications tax reform under perfect competition and imperfect competition.

### 3.1. TECHNOLOGY

Constant returns to scale production is assumed for competitive sectors. Given input price vector $\mathbf{p}$, the unit cost function in competitive sector $i$, $c_i(\mathbf{p})$, is assumed to have the constant elasticity of substitution (CES) form:

$$c_i(\mathbf{p}) = \left[ a_{VA}^i \left( P_{VA}^i \right)^{1-\sigma_i} + \sum_j a_j \left( P_j^i \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}}, \tag{3.1}$$

where $a_j^i$ is intermediate input $j$’s share parameter, $a_{VA}^i$ is the value-added share parameter, $p_j$ is the price of intermediate input $j$, and $\sigma_i$ is the elasticity of substitution between any two inputs. The price of value added, $P_{VA}^i$, is specified as a CES aggregate of the wage rate, $w$, and the rental rate of capital, $r$, gross of taxes:

$$P_{VA}^i = \left( a_{VA}^L w^{1-\sigma_{LK}} + a_{VA}^K \left[ r (1 + t^K_r) \right]^{1-\sigma_{LK}} \right)^{\frac{1}{1-\sigma_{LK}}}, \tag{3.2}$$

where $a_{VA}^L$ and $a_{VA}^K$ are labor and capital’s share parameter in the production of value added of good $i$, $\sigma_{LK}$ is the elasticity of substitution between labor and capital, and $t^K_r$ is the tax rate on capital.

A representative firm in the imperfectly competitive telecommunications sector $T$ has both fixed setup costs and variable costs. The fixed setup costs for the telecommunications sector $T$ are given by

$$F_T(w, r) = r \left( 1 + t^K_T \right) F^K_T + w F^L_T, \tag{3.3}$$
where $F^L_T$ is the fixed input of labor to the telecommunications industry, and $F^K_T$ is the fixed input of capital to the industry. The firm’s variable costs have the same form as Equation 3.1. If the sector produces $y_T$ units of output, then average costs, $ac_T$, are given by

$$ac_T(y_T) = c_T(p) + \frac{F_T(w,r)}{y_T}.$$  

(3.4)

Figure 1 illustrates the cost assumptions for the representative firm in the imperfectly competitive telecommunications sector.

3.2. PREFERENCES

Because the analysis focuses on efficiency, final demand (including investment and government demand) is generated by a single representative consumer. The representative consumer’s utility is used to compute an equivalent variation measure of efficiency. The consumer’s utility is the CES aggregate:
where \( l \) is the representative consumer’s consumption of leisure, \( \theta_l \) and \( \theta_c \) are the expenditure shares for leisure and consumption goods, \( \sigma_l \) is the elasticity of substitution between consumption and leisure, and \( C \) is composite consumption:

\[
C = \left( \sum_i \theta_i C_i \right)^{\frac{1}{\rho}},
\]

where \( \theta_i \) is the representative consumer’s expenditure share for good \( i \), \( \Sigma \theta_i = 1 \), and \( \sigma_c \) is the elasticity of substitution between consumption goods. Following the Spence (1976) and Dixit and Stiglitz (1977) formulation for monopolistic competition, the representative consumer substitutes between the \( N \) differentiated goods of imperfectly competitive sector \( T \) according to a CES subutility function:

\[
U(C_T) = \left( \theta_l \frac{\sigma_{l-1}}{\sigma_l} + \theta_c C \frac{\sigma_{l-1}}{\sigma_c} \right)^{\frac{\sigma_{l-1}}{\sigma_l}},
\]

where \( \rho \) is a measure of the consumer’s taste for variety, and \( \beta = 1 / (1 - \rho) \) is the elasticity of substitution between any two differentiated products of the sector.

The representative consumer receives income from selling labor, \( L \), and capital, \( K \), and from government transfers, \( G \). The consumer’s budget constraint is

\[
\sum_i p_i C_i = wL + rK + G.
\]

Given total time endowment, \( \tilde{t} \), the consumer maximizes utility subject to the budget constraint and the time constraint

\[
L = \tilde{t} - l,
\]
to yield equilibrium demand for goods.

3.3. GOVERNMENT

The government budget constraint is

$$G = \sum_i t^C_i p_i C_i + r \sum_i t^K_i K_i,$$

where $t^C_i$ is the commodity tax rate on good $i$. The government returns all revenues to the representative consumer.

3.4. EQUILIBRIUM CONDITIONS

3.4.1. Market Clearance

A general equilibrium exists when prices are such that all goods and factor markets clear. Market clearance is satisfied by the following goods and factor market equations:

$$y_i = \sum_j a_{ij} y_j + C_i,$$

$$(L_i + F^L_i) = L,$$

$$(K_i + F^K_i) = K,$$

where $y_i$ is the total output of industry $i$, $a_{ij}$ is the intermediate input requirements of $j$ in the production of good $i$, $F^L_i$ is the fixed input of labor to industry $i$, and $F^K_i$ is the fixed input of capital to sector $i$.

3.4.2. Price-Cost Balance

A producer in a constant returns to scale, competitive sector $i$ sets price equal to marginal cost, $c_i(p)$,

$$p_i = c_i(p),$$

(3.14)
and the zero-profit condition ensures that revenues just cover the gross of tax cost of production.

Price for the telecommunications services of representative firm $i$ in the imperfectly competitive telecommunications sector, $T$, is given by (see Appendix A)

$$ p_T = \frac{c_T(p)}{\beta} $$

That is, price is a constant markup over marginal costs, $c_T(p)$. A composite price for the sector, $p_T$, is the CES aggregate:

$$ p_T = \left( \sum_{i} p_{Ti}^{1/\beta} \right)^{-\beta}. $$

### 3.5. EXISTENCE AND UNIQUENESS

The nonconvexities of setup costs in the telecommunications sector violate the conditions that guarantee the existence and uniqueness of a general equilibrium in an Arrow-Debreu competitive economy. Nevertheless, Negishi (1961) proves existence of a general equilibrium for models with monopolistic competition. The markup formulation used here for the imperfectly competitive sector is a generalization of the Negishi (1961) approach and guarantees the existence of a general equilibrium.

Although uniqueness is not guaranteed for the model used here, I test each equilibrium for uniqueness by solving the model with alternative starting values. No case of multiple equilibria was found.

### 4. CALIBRATION

Appendix B presents the benchmark data for Georgia and gives a detailed description of how the data were compiled. The calibration process selects parameter values such that solving the model reproduces the benchmark data set. Table 1 shows the values of exogenous parameters that are used to calibrate the model. The specification of exogenous parameters relies on literature estimates.
One important parameter is the elasticity of scale, $\varepsilon$. As I show in Appendix A, under a few fairly plausible assumptions, the elasticity of scale is the inverse of the substitution parameter for the differentiated products of the monopolistically competitive sector. Values of the elasticity of scale greater than 1 indicate the presence of economies of scale in an industry. Studies estimate the elasticity of scale for the telecommunications industry to be between 1.04 and 1.16 when technological change is exogenous and higher when technological change is endogenous. For the central case results, I use an elasticity of scale value of 1.1.

5. RESULTS

5.1. TAX REFORMS

This section presents the results of three tax-reform experiments that are described in Table 2. The first tax-reform experiment (TR1) replaces the telecommunications capital tax differential with a uniform capital tax on all sectors. The second tax-reform experiment
(TR2) replaces the capital tax differential with a uniform commodity tax. The third tax-reform experiment (TR3) replaces the telecommunications capital tax differential with a differential commodity tax on the industry.

Table 3 shows the efficiency gains for the tax-reform experiments under perfect competition and imperfect competition and for various differential capital tax rates. The efficiency gains are expressed in marginal terms, that is, as efficiency gains per additional dollar of revenue. For the perfect competition case, the calculated marginal efficiency gains for TR1 and TR2 are the same. The marginal efficiency gains range from 7¢ when the differential capital tax rate is 10% to 21¢ when the rate is 30%. These marginal efficiency gains are of the same orders of magnitude as the marginal efficiency costs calculated elsewhere for capital taxes at the industry level for similar values of the parameters. For example, Ballard, Shoven, and Whalley (1985) found that the marginal efficiency costs for a 1% increase in capital taxes at the industry level is about 22¢ when the existing rate of the capital tax is about 50% (of gross-of-tax capital income). For the imperfect competition case, the calculated marginal efficiency gains for TR1 and TR2 are again roughly the same, but they are now 1½ to 2 times higher than for the perfect competition case, ranging from 19¢ to 35¢ for each additional dollar of revenue.

That the marginal efficiency gains for TR1 and TR2 are the same for the perfect competition case is not surprising. An implication of Diamond and Mirrlees (1971) is that uniform taxation of inputs minimizes the efficiency losses of taxation. And because of the choice of a homothetic utility function with separability between commodities and leisure for the preferences of the representative consumer, uni-

<table>
<thead>
<tr>
<th>Tax Reform</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>TR2</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>TR3</td>
<td>0</td>
<td>−0.03</td>
<td>−0.10</td>
<td>0.09</td>
<td>0.07</td>
<td>−0.01</td>
</tr>
</tbody>
</table>
form commodity taxation also minimizes efficiency losses. The equality of the efficiency gains of TR1 and TR2 for the imperfect competition case also makes sense. Because of the zero-profit condition in the model of imperfect competition used here, the only indirect efficiency effect is due to the output repression effect mentioned in Section 2. Recall that the output repression effect is the additional efficiency losses that arise because of a tax-induced reduction in output when price is initially above marginal costs. Because both tax reforms eliminate the tax-induced reduction in the telecommunications industry output, the efficiency gains are the same.

Under perfect competition, the calculated marginal gains from replacing the differential telecommunications capital tax with an equal-yield differential commodity tax (TR3) range from 0 when the capital tax differential is 10% to marginal efficiency costs of 10¢ when the rate is 30%. For the imperfect competition case, the calculated marginal efficiency gains of TR3 range from 9¢ when the capital tax differential is 10% to marginal efficiency costs of 1¢ when it is 30%.

There are several interesting aspects to the results for TR3. First, comparing the results for TR3 to that for TR1 and TR2 suggests that uniform taxation is more desirable than differential taxation, whether the tax instrument is a capital tax or a commodity tax. Second, the direction of the welfare change is sensitive to market structure and the tax rate. Thus, if we assume perfect competition for the telecommunications industry, replacing the differential capital tax with a differential commodity tax generates efficiency losses. But if we assume that the telecommunications industry is imperfectly competitive, then replacing the differential capital tax with a differential commodity tax generates small efficiency gains if the differential capital tax rate is not too high.

Finally, as a follow-up on the last point, that efficiency losses are incurred when the differential capital tax is replaced with a differential commodity tax in the perfect competition case suggests that the direct efficiency costs of the differential commodity tax is higher than the direct efficiency costs of the differential capital tax. Yet, for the imperfect competition case, there are small net efficiency gains when the differential capital tax is replaced with a differential commodity tax, at least at the lower tax rates. This suggests that the additional efficiency
losses due to imperfect competition are much higher for the capital tax than for the commodity tax and, for the lower tax capital differentials, overwhelm the direct efficiency costs. This is a typical second-best result in which a policy instrument that has an efficiency advantage over alternative policy instruments in a first-best setting may lose its advantage in the second-best setting in which an initial market distortion is taken into account.5

### 5.2. SENSITIVITY ANALYSIS

A question that comes to mind immediately is the extent to which the results presented here are robust to the choice of the calibration parameters. To answer this question, I conduct sensitivity analyses on key parameters of the model for the three tax reforms. Tables 4 through 6 show the results of the sensitivity analyses. First, I explore

---

**TABLE 4: Sensitivity Analysis on Select Parameters (First Tax-Reform Experiment—TR1)**

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Competition</td>
<td>Imperfect Competition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Labor-capital substitution elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_{LK} = 0$</td>
<td>0.05</td>
<td>0.09</td>
<td>0.13</td>
<td>0.16</td>
<td>0.21</td>
<td>0.26</td>
</tr>
<tr>
<td>b. $\sigma_{LK} = 0.6$</td>
<td>0.06</td>
<td>0.12</td>
<td>0.18</td>
<td>0.18</td>
<td>0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>c. $\sigma_{LK} = 1.2$</td>
<td>0.07</td>
<td>0.15</td>
<td>0.23</td>
<td>0.19</td>
<td>0.28</td>
<td>0.36</td>
</tr>
<tr>
<td>2. Uncompensated labor-supply elasticity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\varepsilon_L = 0$</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>b. $\varepsilon_L = 0.3$</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>c. $\varepsilon_L = 1.5$</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.19</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>3. Consumption of elasticity of substitution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_c = 0$</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>b. $\sigma_c = 0.6$</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.12</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>c. $\sigma_c = 1.2$</td>
<td>0.08</td>
<td>0.16</td>
<td>0.25</td>
<td>0.23</td>
<td>0.32</td>
<td>0.39</td>
</tr>
<tr>
<td>4. Elasticity of scale</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\varepsilon = 1.05$</td>
<td></td>
<td></td>
<td></td>
<td>0.11</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>b. $\varepsilon = 1.08$</td>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>c. $\varepsilon = 1.13$</td>
<td></td>
<td></td>
<td></td>
<td>0.24</td>
<td>0.33</td>
<td>0.39</td>
</tr>
</tbody>
</table>
the effect of the labor-capital substitution elasticity on the model results. The greater the ease of substituting labor for capital, the higher the efficiency losses we might expect from a differential capital tax because of the greater change in capital use when the tax is imposed. Therefore, the higher the labor-capital substitution elasticity, the greater the efficiency gains we expect from a reform that eliminates the tax. Rows 1a through 1c of each table show that as expected, the efficiency gains (losses) of the tax reform increase (decrease) with increasing values of the labor-capital substitution elasticity.

The labor-supply elasticity might be expected to have only an indirect effect on the efficiency costs of a capital tax. A larger labor-supply elasticity means greater labor market flexibility. And greater labor market flexibility makes it easier for other industries to absorb labor and capital that are shifted away from the differentially higher taxed industry, increasing the capital supply response to the tax. But the

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Marginal Efficiency Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perfect Competition</td>
</tr>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>1. Labor-capital substitution elasticity</td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_{LK} = 0$</td>
<td>0.04</td>
</tr>
<tr>
<td>b. $\sigma_{LK} = 0.6$</td>
<td>0.06</td>
</tr>
<tr>
<td>c. $\sigma_{LK} = 1.2$</td>
<td>0.07</td>
</tr>
<tr>
<td>2. Uncompensated labor-supply elasticity</td>
<td></td>
</tr>
<tr>
<td>a. $\varepsilon_{L} = 0$</td>
<td>0.07</td>
</tr>
<tr>
<td>b. $\varepsilon_{L} = 0.3$</td>
<td>0.07</td>
</tr>
<tr>
<td>c. $\varepsilon_{L} = 1.5$</td>
<td>0.07</td>
</tr>
<tr>
<td>3. Consumption of elasticity of substitution</td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_{c} = 0$</td>
<td>0.02</td>
</tr>
<tr>
<td>b. $\sigma_{c} = 0.6$</td>
<td>0.05</td>
</tr>
<tr>
<td>c. $\sigma_{c} = 1.2$</td>
<td>0.08</td>
</tr>
<tr>
<td>4. Elasticity of scale</td>
<td></td>
</tr>
<tr>
<td>a. $\epsilon = 1.05$</td>
<td></td>
</tr>
<tr>
<td>b. $\epsilon = 1.08$</td>
<td></td>
</tr>
<tr>
<td>c. $\epsilon = 1.13$</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5: Sensitivity Analysis on Select Parameters (Second Tax-Reform Experiment—TR2)
The magnitude of this effect depends on the relative factor intensity between the differentially taxed telecommunications industry and other industries. In fact, as rows 2a through 2c of Tables 4 through 6 show, the uncompensated labor-supply elasticity has little or no impact on the efficiency gains or losses of the tax reforms.

A differential capital tax on the telecommunications industry increases the price of telecommunications services relative to other goods. Greater ease of substituting other goods for the relatively more expensive telecommunications output therefore causes greater efficiency losses from the differential capital tax. Consequently, we might expect any tax reform that replaces or eliminates the tax differential to be sensitive to the consumption elasticity of substitution. Moreover, we might expect the efficiency response to be more sensitive to the consumption elasticity of substitution for a reform that eliminates the tax differential (uniform taxation) than for a reform that replaces one

---

**TABLE 6: Sensitivity Analysis on Select Parameters (Third Tax-Reform Experiment—TR3)**

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Marginal Efficiency Gains</th>
<th>Perfect Competition</th>
<th>Imperfect Competition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perfect Competition</td>
<td>Imperfect Competition</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>1. Labor-capital substitution elasticity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_{LK} = 0$</td>
<td>$-0.03$</td>
<td>$-0.07$</td>
<td>$-0.16$</td>
</tr>
<tr>
<td>b. $\sigma_{LK} = 0.6$</td>
<td>$-0.01$</td>
<td>$-0.02$</td>
<td>$-0.13$</td>
</tr>
<tr>
<td>c. $\sigma_{LK} = 1.2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-0.09$</td>
</tr>
<tr>
<td>2. Uncompensated labor-supply elasticity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\epsilon_L = 0$</td>
<td>$0$</td>
<td>$-0.03$</td>
<td>$-0.10$</td>
</tr>
<tr>
<td>b. $\epsilon_L = 0.3$</td>
<td>$0$</td>
<td>$-0.03$</td>
<td>$-0.10$</td>
</tr>
<tr>
<td>c. $\epsilon_L = 1.5$</td>
<td>$0$</td>
<td>$-0.03$</td>
<td>$-0.10$</td>
</tr>
<tr>
<td>3. Consumption of elasticity of substitution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\sigma_c = 0$</td>
<td>$0$</td>
<td>$-0.02$</td>
<td>$-0.07$</td>
</tr>
<tr>
<td>b. $\sigma_c = 0.6$</td>
<td>$0$</td>
<td>$-0.02$</td>
<td>$-0.09$</td>
</tr>
<tr>
<td>c. $\sigma_c = 1.2$</td>
<td>$0$</td>
<td>$-0.02$</td>
<td>$-0.11$</td>
</tr>
<tr>
<td>4. Elasticity of scale</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $\epsilon = 1.05$</td>
<td>$0.04$</td>
<td>$-0.02$</td>
<td>$-0.05$</td>
</tr>
<tr>
<td>b. $\epsilon = 1.08$</td>
<td>$0.07$</td>
<td>$0.05$</td>
<td>$-0.03$</td>
</tr>
<tr>
<td>c. $\epsilon = 1.13$</td>
<td>$0.12$</td>
<td>$0.11$</td>
<td>$0.01$</td>
</tr>
</tbody>
</table>
differential tax instrument with another. Rows 3a through 3c of Tables 4 and 5 show that the efficiency effects are indeed sensitive to the consumption elasticity of substitution for the reforms that replace the tax differential with uniform taxation. The corresponding rows in Table 6 show that when the telecommunications capital tax is replaced with a telecommunications commodity tax, the consumption elasticity of substitution has little or no effect on the efficiency effects of the tax reform.

Finally, I investigate the sensitivity of the results to the elasticity of scale. As explained in Section 2, in the presence of imperfect competition, a capital tax generates additional efficiency losses beyond its direct efficiency costs. For the model of imperfect competition used here, the additional efficiency losses are primarily due to the output repression effect described in Section 2 (the zero-profit condition of the monopolistic competition model ensures this). Recall that the output repression effect arises because when price is initially above marginal cost, any tax-induced reduction of output generates efficiency losses in addition to the direct efficiency costs of the tax. The elasticity of scale is a measure of the degree to which the telecommunications output price exceeds marginal costs; that is, it measures the degree to which the industry is monopolistic. The efficiency effects for a tax reform that eliminates or reduces the tax distortion should therefore be sensitive to the elasticity of scale. Indeed, rows 4a through 4c of Tables 4 through 6 show that the efficiency gains (losses) of the tax reforms increase (decrease) with increasing values of the elasticity of scale. That is to say, the more monopolistic the industry, the greater the efficiency gains from eliminating or replacing the capital tax differential.

6. CONCLUSIONS

The results presented here compare the relative efficiency of some alternative indirect tax structures to differential capital taxation of the telecommunications sector. Under perfect competition, the marginal efficiency gains from shifting from differential capital taxation to uniform taxation of the telecommunications sector are substantial. They are the same orders of magnitude as the marginal efficiency costs calculated elsewhere in the literature for capital taxes at the industry
level. But due to the presence of large setup costs, the telecommunications sector product market is likely to be characterized by imperfect competition. When account is taken of imperfect competition in the sector’s product market, the marginal efficiency gains are even larger. They are on the order of twice that obtained for the perfect competition case.

The results of this study offer a few lessons for tax policy design in a deregulated telecommunications marketplace and for tax policy design in general. The results suggest that uniform taxation of the telecommunications sector is desirable whether the industry is competitive or imperfectly competitive. The results also indicate that uniform taxation of the sector is desirable whether the tax instrument is a capital tax or a commodity tax. By contrast, the relative efficiency of a tax instrument under differential taxation of the industry depends on the market structure. This last result highlights the importance of taking into account indirect efficiency losses that arise under imperfect competition and have broader implications for empirical measurements of the efficiency costs of taxes in general. The results presented here show that under imperfect competition, the second-order indirect efficiency losses may overwhelm what we often think of as the first-order direct efficiency costs. Therefore, a tax instrument that is relatively efficient under perfect competition may be relatively inefficient under imperfect competition. Thus, in evaluating the efficiency costs of a tax structure, greater attention ought to be paid to market structure because it can crucially affect efficiency costs.

One limitation of the research presented here is that the model is static. Because the model is static, capital in the model is inelastically supplied; the calculated efficiency gains from eliminating the capital tax differential thus miss efficiency gains that arise from reduced distortions in the savings decision. In a dynamic version of the model, the greatest changes would likely occur in the relative efficiency of alternative tax instruments. For example, a reform that replaced the capital tax differential with a uniform commodity tax would probably realize higher efficiency gains than one that replaced the capital tax differential with a uniform capital tax, as the uniform commodity tax would likely distort the savings decision less than the uniform capital tax.
APPENDIX A
Equilibrium in the Imperfectly Competitive Telecommunications Sector

For imperfect competition in the telecommunications sector’s product market, I assume that the pricing behavior of firms follows the monopolistic competition model of Spence (1976) and Dixit and Stiglitz (1977). Formally, demand for the differentiated outputs, \( Y_{Ti} \), of the increasing returns to scale sector is derived from a constant elasticity of substitution subutility function of the following form:

\[
Y_T = \left( \frac{\sum_{i} Y_{Ti}^{\rho \beta}}{\rho} \right)^{1/\rho},
\]

(A.1)

where \( \rho < 1 \) for concavity, and \( \beta = 1 / (1 - \rho) \) is the elasticity of substitution between any two differentiated products of the industry. From the first-order conditions for maximization of subutility function \( Y_T \), the demand for differentiated output \( Y_{Ti} \) is

\[
Y_{Ti} = \left( \frac{p_T}{\sum_{j=1}^{N} p_{Tj}^{1-\beta}} \right)^{\beta} E_T,
\]

(A.2)

where \( E_T \) is total expenditure on the differentiated products of the sector, and \( p_{Ti} \) and \( p_{Tj} \) are the price of differentiated products \( i \) and \( j \), respectively. If \( p_T \) denotes the price of the composite product \( Y_T \) of the monopolistically competitive sector, then,

\[
p_T = \left( \sum_{i} p_{Ti}^{1/\beta} \right)^{\beta}.
\]

(A.3)

The price, \( p_{Ti} \), received by the representative firm can be viewed as the value of the marginal product of \( Y_{Ti} \) in producing \( Y_T \):

\[
p_{Ti} = p_T (1 / \rho) Y_T^{1-\rho} Y_{Ti}^{\rho - 1}.
\]

(A.4)

Thus, the revenues, \( R_{Ti} \), of the representative firm is

\[
R_{Ti} = p_{Ti} Y_{Ti} = \left[ p_T Y_T^{1-\rho} \right] Y_{Ti}^{\rho}.
\]

(A.5)

Here, I make the assumptions that (a) firms act in a Bertrand-Nash fashion, taking their competitor’s price as fixed, and (b) each firm views itself as small in the market. Thus, \( Y_T \) and \( p_T \) are constant. The bracketed term in Equation A.5 is therefore a constant. Marginal revenues for the representative firm is therefore
Now, recall that the total costs comprise fixed costs plus constant variable costs, \( c_T \). Free entry results in zero profits so that price equals average cost, \( ac_T \). Thus, equating marginal revenues to marginal costs and price to average costs gives the following:

\[ p_T \beta = c_T, \]

\[ p_T = ac_T. \]  

(A.7)

Furthermore, by the definition of elasticity of scale,

\[ \varepsilon_T = \frac{ac_T}{c_T} \]  

(A.8)

Using this definition with Equations A.7 gives

\[ \varepsilon_T = \frac{1}{\beta} \]  

(A.9)

I can therefore use elasticity of scale estimates for the telecommunications sector from engineering studies to calibrate the sector under the monopolistic competition assumption.

**APPENDIX B**

**The Benchmark Data Set:**

**A Social Accounting Matrix (Sam) for Georgia**

The balanced benchmark data for the computable general equilibrium model is constructed in the form of a regional SAM for Georgia. A SAM is a traditional input-output (I-O) table extended to include the financial flows in an economy for a given year. The financial-flows data (savings/dis-savings, budget-deficit/surplus, balance of payments, etc.) are included mainly to serve as balancing elements as described in the previous section. The main source of data for constructing the SAM is micro IMPLAN (IMpact analysis for PLANning) 1992, first developed by the USDA Forest Service (Olson, Lindall, and Maki 1993) and now produced and sold by MIG, Inc. It comprises data and software for aggregating its 528 sectors and generating regional I-O accounts and models. To construct the SAM from the IMPLAN I-O accounts, I used the I-O consistent financial-flows data for Georgia, also supplied by MIG, Inc.
B.1. SECTOR AGGREGATION

The 528 sectors were aggregated into the five industries shown in Table B1. Table B2 shows the sectors that are not aggregated.

B.2. VALUE ADDED

Value-added data are derived from IMPLAN Lister Report No. 404A, which includes employee compensation, proprietary income, other property income, and indirect business taxes. Employee compensation comprises total payroll costs (wages or salaries, and benefits) paid by industries. Indirect business taxes are sales, excise, property, and other taxes paid in the normal course of doing business. It does not include taxes paid on net income. Proprietary income is income from self-employment. Other property income includes corporate income, rental income, interest, and corporate transfer payments.

Indirect taxes are subtracted from total value added and appropriate this to government. The remaining total value added are distributed to factors as follows. All employee compensation are attributed to labor, all other property income to capital, and proprietary income between labor (29.2%) and capital (70.8%). This ratio is derived from the more detailed data available in the complementary financial-flows data.
B.3. INTERMEDIATE DEMAND

Intermediate demand is represented in the SAM by a regional (state) interindustry transaction submatrix. One piece of the data for this submatrix is obtained from Lister Report No. 402, which records each industry’s purchases of goods and services from other local industries. The row sums of this report therefore represent locally produced regional intermediate demand by industry. To complete the interindustry transaction submatrix, the demand for imported intermediate inputs by industries in the region is included. These data are obtained by adding the data from Lister Report No. 112 (Regional Competitive Imports to Intermediate Demand) to the data from Lister Report No. 108 (Regional Noncompetitive Imports to Intermediate Demand).

B.4. REGIONALLY PRODUCED COMMODITY DEMAND.

Regional consumption demands are obtained from Lister Reports No. 403A and No. 403B. Lister Report No. 403A includes household and government consumption. Households are classified according to income as low, medium, and high, but these are aggregated into the consumption of a representative consumer because efficiency is the main concern of the analysis. Similarly, the consumption data for government is classified by level into state or local, and federal. Again, these are aggregated into consumption of a single public good used by the representative consumer.

Lister Report No. 403B contains data on regional trade and investment demand. Investment demand comprises inventory additions and capital formation, whereas trade demand comprise the state’s foreign and domestic exports by industry.

B.5. IMPORTED COMMODITY DEMAND

Domestic and foreign imports data are obtained from IMPLAN’s Lister Report No. 113A (Regional Competitive Imports to Consumption Demand) and Lister Report No. 110A (Regional Noncompetitive Imports to Consumption Demand), which includes data, and household and government consumption of imports. IMPLAN’s Lister Reports No. 110B and No. 113B provide comparable data for investment and trade import demand.

B.6. FINANCIAL FLOWS

The financial-flows data give a detailed breakdown of taxation, income, and savings data for government and households and also serve as a check for the IMPLAN
I-O data. MIG, Inc., derives these data primarily from the 1990 BLS Consumer Expenditure Survey. Certain items in the accounts serve as balancing items when the row-and-column-sum method is used to balance the SAM. For example, the representative household receives income from selling factors in the factor markets and from government transfers. The income is used to purchase goods, pay taxes, and save. The balancing procedure uses the convention that any discrepancies between spending and income is savings or dis-savings. Similarly, any difference between government revenues and spending is a surplus or deficit. Foreign-trade balance occurs when the difference between the value of imports and exports equals net capital inflows. In the end, these financial data are aggregated and enter the model as lump-sum exogenous income to the representative consumer and the government, who returns it all to the representative consumer.

Table B3 presents the final balanced 1992 SAM for the Georgia economy. All quantities are in terms of millions of 1992 dollars.

### TABLE B3: 1992 Georgia Social Accounting Matrix (in millions of dollars)

<table>
<thead>
<tr>
<th>Expenditures</th>
<th>Agr</th>
<th>Min</th>
<th>Mfg</th>
<th>Srv</th>
<th>Com</th>
<th>Lab</th>
<th>Cap</th>
<th>Hh</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agr</td>
<td>141</td>
<td>61</td>
<td>2,075</td>
<td>146</td>
<td>0</td>
<td>347</td>
<td>2,225</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>39</td>
<td>65</td>
<td>654</td>
<td>3,260</td>
<td>258</td>
<td>14,298</td>
<td>1,646</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mfg</td>
<td>164</td>
<td>1,430</td>
<td>10,601</td>
<td>2,791</td>
<td>93</td>
<td>11,916</td>
<td>53,914</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Srv</td>
<td>486</td>
<td>4,355</td>
<td>10,043</td>
<td>23,845</td>
<td>280</td>
<td>65,777</td>
<td>33,041</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Com</td>
<td>3</td>
<td>32</td>
<td>89</td>
<td>439</td>
<td>287</td>
<td>1,353</td>
<td>4,263</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lab</td>
<td>889</td>
<td>5,451</td>
<td>17,706</td>
<td>51,871</td>
<td>2,241</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cap</td>
<td>1,578</td>
<td>2,833</td>
<td>9,561</td>
<td>25,029</td>
<td>2,099</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hh</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>78,157</td>
<td>41,098</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ind Tax</td>
<td>24</td>
<td>58</td>
<td>692</td>
<td>4,000</td>
<td>168</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cap Tax</td>
<td>208</td>
<td>400</td>
<td>2,198</td>
<td>9,479</td>
<td>509</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2,020</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agr</td>
<td>720</td>
<td>42</td>
<td>1,776</td>
<td>121</td>
<td>0</td>
<td>465</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>8</td>
<td>216</td>
<td>490</td>
<td>1,401</td>
<td>41</td>
<td>213</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mfg</td>
<td>630</td>
<td>4,602</td>
<td>23,023</td>
<td>6,605</td>
<td>165</td>
<td>24,659</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Srv</td>
<td>104</td>
<td>649</td>
<td>1,966</td>
<td>8,839</td>
<td>326</td>
<td>18,956</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Com</td>
<td>2</td>
<td>25</td>
<td>36</td>
<td>1</td>
<td>0</td>
<td>1,028</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5,268</td>
<td>20,219</td>
<td>80,909</td>
<td>137,827</td>
<td>6,467</td>
<td>78,157</td>
<td>41,098</td>
<td>139,012</td>
<td>97,108</td>
</tr>
</tbody>
</table>

I-O data. MIG, Inc., derives these data primarily from the 1990 BLS Consumer Expenditure Survey. Certain items in the accounts serve as balancing items when the row-and-column-sum method is used to balance the SAM. For example, the representative household receives income from selling factors in the factor markets and from government transfers. The income is used to purchase goods, pay taxes, and save. The balancing procedure uses the convention that any discrepancies between spending and income is savings or dis-savings. Similarly, any difference between government revenues and spending is a surplus or deficit. Foreign-trade balance occurs when the difference between the value of imports and exports equals net capital inflows. In the end, these financial data are aggregated and enter the model as lump-sum exogenous income to the representative consumer and the government, who returns it all to the representative consumer.

Table B3 presents the final balanced 1992 SAM for the Georgia economy. All quantities are in terms of millions of 1992 dollars.

### NOTES
1. See, for example, Ballard, Shoven, and Whalley (1985).
2. See Willig (1983) and Section 2.

REFERENCES


Fitzroy A. Lee is an assistant professor in the Department of Economics in the Faculty of Liberal Arts and Sciences at Tulane University. He is currently on leave from Tulane University, working as Director of Revenue Estimation and Tax Research at the Office of Research and Analysis in the Office of the Chief Financial Officer for the District of Columbia. His research interests include the equity-efficiency tradeoff aspects of tax policy and the effect of globalization and the Internet on tax policy.
The directional distance function provides a complete characterization of the production technology and, when differentiable, can be used to derive shadow prices for nonmarket outputs. A quadratic functional form and the linear programming least absolute deviations method is used to implement the model and to estimate shadow prices and total value for the nonmarket characteristics of Missouri conservation land.

SHADOW PRICES OF MISSOURI
PUBLIC CONSERVATION LAND

ROLF FÄRE
SHAWNA GROSSKOPF
Oregon State University

WILLIAM L. WEBER
Southeast Missouri State University

1. INTRODUCTION

In 1976, after a signature gathering initiative petition, Missourians passed a constitutional amendment for an addition to the state sales tax, with the proceeds earmarked for conservation use (State of Missouri 1977-1978). With slightly more than 1.7 million votes cast, the amendment passed with just 50.8% of the votes in favor. Carefully watched by other states, passage of the amendment put Missouri first in per capita conservation outlays (Renken 1976). In the years since its passage, the 1/4 of 1% sales tax has enabled the Missouri Department of Conservation to add to and maintain more than 700,000 acres of public conservation lands in 114 counties (Zekor 1996). Today, these conservation lands include forests and natural areas, critical wildlife habitats, unique ecosystems, springs, former farmlands, river and stream accesses, and public lakes. In 1994, the Missouri Department of Conservation spent more than 5 million dollars making additions to 16 existing conservation areas and acquiring 13,000 acres of forest land and three public river access sites (“A Summary” 1995). From 1976 to 1994, the conservation sales and use tax revenues grew from $20 mil-
lion to more than $64 million (Missouri Department of Revenue 1994).

In this article, we provide a framework for estimating the shadow prices and total value of the nonmarket or public good characteristics of Missouri conservation land. When a consumer chooses a bundle of goods and services subject to a given technology, the first order (equilibrium) conditions imply that the consumer’s marginal rate of substitution is equal to the producer’s marginal rate of transformation. Thus, one can infer the shadow price of nontraded goods from either the consumer or the production relationship (Färe and Grosskopf 1998). Here, we exploit the directional output distance function to derive shadow prices and total value of Missouri conservation land. In particular, we formalize and estimate a quadratic form of this function and use its derivatives to compute shadow prices (Chambers, Chung, and Färe 1996, 1998).

The pricing of nontradeable goods and services, such as environmental amenities or other public goods, is important for a variety of reasons. First, it is widely known that money income does not accurately measure consumer well-being when utility depends on nonmarket goods and services. Because of this, interest has been growing in modifying national income accounts by imputing values of environmental amenities and dis-amenities. Without these adjustments, money income will be a biased measure of well-being depending on the degree of privatization of the environmental amenities. In Britain, where private fee-based hunting and fishing preserves have been the norm for centuries, the degree of bias will be small relative to a country such as the United States, where most hunting and fishing opportunities still occur on public lands and waterways. In addition, certain private averting expenditures, like a water treatment plant, may substitute for an environmental amenity, such as the watershed protection services provided by woodlands. Clearly, water treatment plants show up in national income accounts, whereas the watershed protection value of woodlands does not. Shadow pricing methods could allow “green” accounts to be set up that measure the revenue that could be earned if the resource were privatized and sold in the marketplace.

Second, when actual price data are available, a comparison of shadow prices and actual prices can provide information to policy
makers on the direction of change in the mix of outputs that would enhance economic welfare. For example, Grosskopf et al. (1999) compare actual school district teacher and administrative salaries with shadow prices to determine whether schools are underutilizing or overutilizing teachers relative to administrators. Policy makers may also use shadow price information to establish fees for the public’s use of environmental amenities, or to compare the fees charged with the marginal benefits received as measured by the shadow price. Similarly, policy makers can use shadow price information to help them estimate the dollar amount they may realize from auctioning permits to pollute. Coggins and Swinton (1996) and Swinton (1998) have used shadow price information to compare the actual prices paid for SO\textsubscript{2} permits with the estimated shadow price of a one-unit decline in SO\textsubscript{2}. Their empirical results suggest that shadow prices provide a good approximation to the actual prices paid for SO\textsubscript{2} permits by electric utilities.

Third, shadow price models can help policy makers understand the distributional implications of public expenditure programs when there is no discernible link between taxes paid and benefits received. This is especially important for programs like Missouri’s ½ of 1% conservation sales tax. Unless there is a one-to-one relationship between conservation sales taxes paid by citizens of a county and the marginal benefits received from conservation lands, the tax will be redistributive and may alter the socially efficient level of production.

Fourth, shadow price models can allow economists a further check into estimated measures of willingness to pay from alternative models such as the contingent valuation method (CVM), or capitalization methods commonly used to value the amenities captured by land rents. CVM models in particular have enjoyed recent widespread appeal and criticism for their use in valuing environmental amenities. The most common criticism of the CVM is that people are responding to a survey and not a budget constraint, which tends to bias upward their support of the environmental amenity. Brown and Shogren (1998) point out that the CVM has produced estimates that U.S. households are willing to spend more than 1% of GDP to preserve less than 2% of endangered species, a figure they contend is “suspiciously high.” Because shadow price models examine the actual structure of
production given resource constraints, they should provide a viable check on CVM estimates of willingness to pay.

In the next section, we show how the directional distance function can be used to derive shadow prices. Section 3 provides a functional form that can be used to estimate the directional output distance function. In Section 4, the data are described for implementing a shadow pricing model for the characteristics of Missouri conservation land. The empirical results are also described. The final section offers a summary and conclusion.

2. THE THEORETICAL UNDERPINNING

The shadow pricing model is derived from the directional output distance function. In particular, by applying the envelope theorem to the distance function, shadow prices are obtained.

We denote inputs by \( x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \) and outputs by \( y = (y_1, y_2, \ldots, y_m) \in \mathbb{R}^m \). The technology, here represented by the output set, \( P(x) \), consists of all feasible input-output pairs, \( (x,y) \); that is,

\[
P(x) = \{ y: x \text{ can produce } y \}. \tag{1}
\]

We impose standard assumptions on \( P(x) \), namely, it is a closed, bounded, convex set with inputs and outputs freely disposable.\(^1\) In this article, the output sets are taken to be the counties of Missouri.

Denote the directional output vector by \( g = (g_1, \ldots, g_m) \in \mathbb{R}^m \) and assume \( g \neq 0 \). The associated output distance function is defined as

\[
D_o(x,y; g) = \sup\{ \beta: (y + \beta g) \in P(x) \}. \tag{2}
\]

The directional output distance function scales outputs in a chosen direction, \( g \), to the frontier of \( P(x) \). For decision-making units operating on the frontier of \( P(x) \), the value of the directional output distance function is zero, indicating that no further expansion of outputs in the direction \( g \) is feasible. The directional output distance function takes positive values for inefficient decision-making units operating below the frontier of \( P(x) \).
The directional output distance function is the output version of Luenberger’s (1992) benefit function, and it inherits the properties imposed on the output set \( P(x) \), \( x \in \mathbb{R}^N_+ \). It is also a complete characterization of the technology in the sense of

\[
y \in P(x) \text{ if and only if } D_o^{-}(x,y; g) \geq 0,
\]

that is, \( x \) can produce \( y \) if and only if the distance function is nonnegative.

From its definition, it follows that \( D_o^{-}(x,y; g) \) satisfies the translation property,

\[
D_o^{-}(x,y + \alpha g; g) = D_o^{-}(x,y; g) – \alpha, \ \alpha \in \mathbb{R}.
\]

This additive property corresponds to the homogeneity condition of Shephard’s output distance function.

Next, we provide a relationship between the revenue function and the directional distance function, \( D_o^{-}(x,y; g) \). The revenue function is defined as

\[
R(x,p) = \max_y \{py: y \in P(x)\},
\]

where \( p = (p_1, \ldots, p_M) \in \mathbb{R}^M_+ \) is the output price vector. The revenue function, \( R(x,p) \), gives the largest feasible revenue that can be obtained from inputs \( x \) and output prices \( p \). Now because \( y + D_o^{-}(x,y; g) \cdot g \) is a feasible output vector, we have the following relationship between the revenue function and the directional distance function:

\[
R(x,p) \geq p[y + D_o^{-}(x,y; g) \cdot g],
\]

or

\[
R(x,p) \geq py + D_o^{-}(x,y; g)pg.
\]

This relationship may also be written as

\[
D_o^{-}(x,y; g) \leq [R(x,p) – py] / pg.
\]
One can also show that the distance function in Equation 7 can be recovered from the revenue function by

$$D_0^{-}(x, y; g) = \inf_p \{ [R(x, p) - py]/pg \}. \quad (8)$$

Applying the envelope theorem to Equation 8 yields our shadow price model:

$$\nabla_y D_0^{-}(x, y; g) = -p/pg. \quad (9)$$

and for two outputs $m$ and $m'$, we have

$$p_m = p_{m'} (\partial D_0^{-}(x, y; g) / \partial y_m) / (\partial D_0^{-}(x, y; g) / \partial y_{m'}). \quad (10)$$

Thus, if one price, say $p_{m'}$, is known, then all other prices, $p_m$, can be calculated using the directional distance function, $D_0^{-}(x, y; g)$.\(^2\)

In the next section, we discuss the additive quadratic functional form used to estimate the directional distance function as a deterministic frontier. The empirical results for the shadow prices and total value of Missouri conservation land characteristics follow in Section 4.

### 3. A FUNCTIONAL FORM AND ESTIMATION PROCEDURE

Recall that the directional output distance function, $D_0^{-}(x, y; g)$, is defined by adding a directional vector $g$ to $y$ and then scaling $g$. As such, $D_0^{-}(x, y; g)$ has the translation property specified by Equation 4.

To estimate the directional output distance function, we take the directional vector $g = (1, 1, \ldots, 1)$ and hence need not include it in the parameterization. Thus, the directional output distance function takes the (additive) quadratic form:

$$D_0^{-}(x, y; 1) = a_0 + \sum_n a_n x_n + \frac{1}{2} \sum_n a_{nn'} x_n x_{n'} + \sum_m b_m y_m$$

$$+ \frac{1}{2} \sum_m b_{nn'} y_n y_{n'} + \sum_m c_{nm} x_n y_m, \quad (11)$$

with $a_{nn'} = a_{n'n}$ and $b_{nn'} = b_{n'm}$. Moreover, as shown by Chambers (1998), the parameter restrictions, $\sum_n b_n = -1, \sum_n b_{nn'} = 0, m = 1, \ldots, M,$
and \( \sum_m c_{nm} = 0, n = 1, \ldots, N \), ensure that the directional output distance function satisfies the translation property in Equation 4.

To estimate the parameters of Equation 11, we follow the work of Aigner and Chu (1968) and estimate a deterministic frontier using linear programming techniques. The parameters of Equation 11 are chosen to minimize the sum of the distance between the frontier technology and the individual county observations. That is, we choose the parameters \( a_0, a_n, a_{nm}, b_m, b_{mm}, \) and \( c_{nm} \), to minimize

\[
\sum_k [D_o^+ (x^k, y^k; 1) - 0], \tag{12}
\]

subject to

\[
D_o^+ (x^k, y^k; 1) \geq 0, k = 1, \ldots, K, \tag{12a}
\]

\[
\partial D_o^+ (x^k, y^k; 1) / \partial y_m \leq 0, k = 1, \ldots, K, m = 1, \ldots, M, \tag{12b}
\]

\[
\partial D_o^+ (x, y; 1) / \partial x_n \geq 0, n = 1, \ldots, N, \tag{12c}
\]

\[
\sum_m b_m = -1, \sum_m b_{mm} = 0, m = 1, \ldots, M, \text{ and } \sum_m c_{nm} = 0, n = 1, \ldots, N. \tag{12d}
\]

The restrictions in Equation 12a constrain each county to be on or below the production frontier of \( P(x) \). The restrictions in (12b) constrain the shadow output prices to be nonnegative for each of the \( K \) counties and \( M \) outputs. The restrictions in (12c) constrain the derivatives of the distance function with respect to inputs to be nonnegative at the mean of the data. Finally, the restrictions in (12d) constrain the directional output distance function to satisfy the translation property.

The shadow price of good \( m \) with respect to good \( m' \) is

\[
P_{m} = \frac{\partial D_o^+ (x, y; 1) / \partial y_m}{\partial D_o^+ (x, y; 1) / \partial y_{m'}} = \frac{b_m + \sum_m b_{mm} y_m + \sum_n c_{nm} x_n}{b_{m'} + \sum_m b_{mm} y_m + \sum_n c_{nm} x_n}. \tag{13}
\]

The following section describes the data used to estimate Equation 11, which then can be used to estimate the shadow prices of conservation land via Equation 13.
4. DATA AND EMPIRICAL RESULTS

A number of researchers have employed aggregate output and aggregate input data to analyze the efficiency of countries and states or regions within a country. See, for example, Färe et al. (1994) for a country level study of efficiency and productivity growth, Domazlicky and Weber (1997) for a study of U.S. state efficiency and productivity, and Mullen and Williams (1987) for an analysis of Standard Metropolitan Statistical Area productivity growth. The model employed here disaggregates further to the county level. We assume that Missouri counties employ the aggregate inputs of labor, capital, and land to produce county personal income and a vector of characteristics of public conservation land located in the county. Labor is the employed civilian labor force in the county, capital is the total property assessed valuation in the county, and land is the number of square miles of land in the county. Income, labor, and land are taken from the CD-ROM U.S. Counties published by the Department of Commerce and are for 1994, the same year as the inventory of Missouri Conservation lands. Total property assessed valuation is taken from the 1994-95 Report of the Public Schools of Missouri (Missouri State Board of Education 1996). Missouri’s Conservation Atlas (Missouri Department of Conservation 1995) reports the number of conservation sites (sites), the total number of acres at each site (acres), and the total acres available for fishing at each site (fish) for each county as of 1994. The atlas also provided additional information on the number of lakes and ponds available at each site. Whereas lake size was always reported, in some instances, pond size was not. In these limited number of cases, ponds were counted as $\frac{1}{2}$ acre in size, the average amount when reported. The sites provide a variety of recreational opportunities including hunting, hiking, fishing, and viewing of wildlife. Sites varied in size from as small as 1 acre to as large as more than 10,000 acres. River access sites tend to be small in size but generally provide the recreationist with miles of fishing and boating opportunities. Therefore, we included both number of sites and number of acres as characteristics of county conservation land. River accesses also provide fishing opportunities for bank fishers, and the atlas reported the number of fishing acres at river accesses. The fishing acres available at river accesses were included with the number of acres of ponds and lakes.
Data on 85 out of 114 Missouri counties were employed. Four counties were dropped because an overlap of conservation areas across adjacent counties made it impossible to determine the share of the area to allocate to each county. Twenty-five other counties were dropped because they produced none of at least one public good attribute.

Descriptive statistics are reported in Table 1 for the 85 counties and for the 72 rural counties and 13 urban counties used in this study. Average county income \( (y_1) \) in 1994 was $1.059 billion. The average county civilian labor force was 25,000 workers \( (x_1) \). County-assessed valuation \( (x_2) \) averaged $439 million, and land area \( (x_3) \) was 609 square miles. The average county also had 7.9 sites \( (y_2) \) containing

<table>
<thead>
<tr>
<th>Variable</th>
<th>County Type</th>
<th>M</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 ) = income ((1,000s))</td>
<td>All</td>
<td>1,059,464.32</td>
<td>3,581,764.40</td>
<td>40,280</td>
<td>29,783,735</td>
</tr>
<tr>
<td>Rural</td>
<td>290,536.15</td>
<td>246,217.21</td>
<td>40,280</td>
<td>1,344,338</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>5,318,143.38</td>
<td>8,139,370.94</td>
<td>358,204</td>
<td>29,783,735</td>
<td></td>
</tr>
<tr>
<td>( y_2 ) = sites</td>
<td>All</td>
<td>7.94</td>
<td>4.99</td>
<td>2</td>
<td>38</td>
</tr>
<tr>
<td>Rural</td>
<td>7.24</td>
<td>3.51</td>
<td>2</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>11.85</td>
<td>9.06</td>
<td>4</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>( y_3 ) = non-fish acres</td>
<td>All</td>
<td>8,049.66</td>
<td>15,969.82</td>
<td>10</td>
<td>130,551</td>
</tr>
<tr>
<td>Rural</td>
<td>8,666.62</td>
<td>17,190.44</td>
<td>10</td>
<td>130,551</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>4,632.64</td>
<td>4,676.62</td>
<td>173</td>
<td>17,278</td>
<td></td>
</tr>
<tr>
<td>( y_4 ) = fish acres</td>
<td>All</td>
<td>288.48</td>
<td>503.20</td>
<td>2</td>
<td>3,255</td>
</tr>
<tr>
<td>Rural</td>
<td>273.62</td>
<td>526.87</td>
<td>2</td>
<td>3,255</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>370.80</td>
<td>348.48</td>
<td>25</td>
<td>1,137</td>
<td></td>
</tr>
<tr>
<td>( x_1 ) = labor</td>
<td>All</td>
<td>25,007.32</td>
<td>68,484.22</td>
<td>895</td>
<td>524,986</td>
</tr>
<tr>
<td>Rural</td>
<td>8,245.24</td>
<td>6,850.30</td>
<td>895</td>
<td>34,817</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>117,843.46</td>
<td>146,732.42</td>
<td>9,770</td>
<td>524,986</td>
<td></td>
</tr>
<tr>
<td>( x_2 ) = capital ((1,000s))</td>
<td>All</td>
<td>439,548.02</td>
<td>1,470,379.03</td>
<td>18,425</td>
<td>12,375,808</td>
</tr>
<tr>
<td>Rural</td>
<td>125,821.31</td>
<td>111,426.13</td>
<td>18,425</td>
<td>598,609</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>2,177,111.36</td>
<td>3,346,546.66</td>
<td>121,359</td>
<td>12,375,808</td>
<td></td>
</tr>
<tr>
<td>( x_3 ) = land () (square miles)</td>
<td>All</td>
<td>609.45</td>
<td>160.11</td>
<td>267</td>
<td>1,179</td>
</tr>
<tr>
<td>Rural</td>
<td>608.82</td>
<td>165.24</td>
<td>267</td>
<td>1,179</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td>612.92</td>
<td>133.73</td>
<td>397</td>
<td>922</td>
<td></td>
</tr>
</tbody>
</table>
8,337 total acres, of which 8,050 acres were land based \( y_3 \), with 288 acres comprising water available for fishing or other water-based activities \( y_4 \) such as canoeing or waterfowl-hunting opportunities. Although rural counties contain more than 90% of all conservation land, the conservation sales tax amendment received a majority in only 12% of rural counties in 1976, while passing in 88% of the urban counties.

To estimate the system of equations given in Equation 12, we first normalized each output and input by its mean value. That is, we redefined outputs, to equal \( y/\text{mean-}\text{y} \) and inputs, \( x \), to equal \( x/\text{mean-x} \). This normalization allows an easy interpretation of the parameter estimates of \( D_o(x,y;1) \). That is, when a county produces outputs equal to the mean value of each output, using inputs equal to the mean value of inputs, county output efficiency is \( D_o(x,y;1) = a_0 + \sum a_{n}n' + \frac{1}{2} \sum \sum b_{mm'} + \frac{1}{2} \sum \sum c_{nn} \).

Table 2 presents the parameter estimates of the directional distance function for the four output–three input model. Given the parameter estimates, the value of \( D_o(x,y;1) \) can be calculated for each county. Counties with \( D_o(x,y;1) = 0 \) operate on the frontier of \( P(x) \) and exhibit no inefficiency. Values of \( D_o(x,y;1) > 0 \) indicate inefficiency. Given the directions \( g = (1, \ldots, 1) \), the value of \( D_o(x,y;1) \) gives the unit expansion in output that could be produced if the given amount of inputs were used efficiently.

The estimates of \( D_o(x,y;1) \) are presented in Table 3. Eighty-five Missouri counties that had nonzero outputs were used to estimate \( D_o(x,y;1) \). County output inefficiency averaged 4.9%, and 13 counties operated on the frontier of \( P(x) \). The mean shadow prices, calculated using each county’s level of inputs and outputs are \( p_2/p_1 = 0.006 \), \( p_3/p_1 = 0.010 \), and \( p_4/p_1 = 0.005 \).

For a hypothetical county with average outputs and average inputs, \( D_o(\bar{x},\bar{y};1) = 0.042 \), indicating 4.2% inefficiency. The shadow prices for the hypothetical county with mean outputs and inputs are \( p_2/p_1 = 0.010 \), \( p_3/p_1 = 0.017 \), and \( p_4/p_1 = 0.005 \). Data envelopment analysis was also used to estimate \( D_o(x,y;1) \) and yielded similar levels of inefficiency.
The shadow prices of the conservation land attributes in terms of income are presented in Table 4. Recall that each output was divided by its mean value to estimate the coefficients of Equation 11, so that a one-unit increase in acres \( (y_3) \), for instance, implies that the number of acres

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>Constant</td>
<td>0.06285</td>
</tr>
<tr>
<td>a₁</td>
<td>( x₁ )</td>
<td>-0.19797</td>
</tr>
<tr>
<td>a₂</td>
<td>( x₂ )</td>
<td>1.07092</td>
</tr>
<tr>
<td>a₃</td>
<td>( x₃ )</td>
<td>0.20040</td>
</tr>
<tr>
<td>a₁₁</td>
<td>( x₁x₁ )</td>
<td>-0.02828</td>
</tr>
<tr>
<td>a₁₂</td>
<td>( x₁x₂ )</td>
<td>0.06434</td>
</tr>
<tr>
<td>a₂₂</td>
<td>( x₂x₂ )</td>
<td>0.79531</td>
</tr>
<tr>
<td>a₂₃</td>
<td>( x₂x₃ )</td>
<td>-0.13057</td>
</tr>
<tr>
<td>a₃₃</td>
<td>( x₃x₃ )</td>
<td>-0.75580</td>
</tr>
<tr>
<td>b₁</td>
<td>( y₁ )</td>
<td>-0.23726</td>
</tr>
<tr>
<td>b₂</td>
<td>( y₂ )</td>
<td>-0.98884</td>
</tr>
<tr>
<td>b₃</td>
<td>( y₃ )</td>
<td>-0.00086</td>
</tr>
<tr>
<td>b₄</td>
<td>( y₄ )</td>
<td>-0.00043</td>
</tr>
<tr>
<td>b₁₁</td>
<td>( y₁y₁ )</td>
<td>-0.00987</td>
</tr>
<tr>
<td>b₁₂</td>
<td>( y₁y₂ )</td>
<td>-0.00023</td>
</tr>
<tr>
<td>b₁₃</td>
<td>( y₁y₃ )</td>
<td>0.00015</td>
</tr>
<tr>
<td>b₁₄</td>
<td>( y₁y₄ )</td>
<td>0.00059</td>
</tr>
<tr>
<td>b₂₂</td>
<td>( y₂y₂ )</td>
<td>-0.00050</td>
</tr>
<tr>
<td>b₂₃</td>
<td>( y₂y₃ )</td>
<td>-0.00012</td>
</tr>
<tr>
<td>b₂₄</td>
<td>( y₂y₄ )</td>
<td>0.00009</td>
</tr>
<tr>
<td>b₃₃</td>
<td>( y₃y₃ )</td>
<td>-0.00057</td>
</tr>
<tr>
<td>b₄₄</td>
<td>( y₄y₄ )</td>
<td>0.00010</td>
</tr>
<tr>
<td>c₁₁</td>
<td>( x₁y₁ )</td>
<td>-0.03934</td>
</tr>
<tr>
<td>c₁₂</td>
<td>( x₁y₂ )</td>
<td>0.01307</td>
</tr>
<tr>
<td>c₁₃</td>
<td>( x₁y₃ )</td>
<td>0.02677</td>
</tr>
<tr>
<td>c₁₄</td>
<td>( x₁y₄ )</td>
<td>-0.00049</td>
</tr>
<tr>
<td>c₂₁</td>
<td>( x₂y₁ )</td>
<td>0.06488</td>
</tr>
<tr>
<td>c₂₂</td>
<td>( x₂y₂ )</td>
<td>-0.02264</td>
</tr>
<tr>
<td>c₂₃</td>
<td>( x₂y₃ )</td>
<td>-0.04327</td>
</tr>
<tr>
<td>c₂₄</td>
<td>( x₂y₄ )</td>
<td>0.00103</td>
</tr>
<tr>
<td>c₃₁</td>
<td>( x₃y₁ )</td>
<td>-0.00892</td>
</tr>
<tr>
<td>c₃₂</td>
<td>( x₃y₂ )</td>
<td>0.00117</td>
</tr>
<tr>
<td>c₃₃</td>
<td>( x₃y₃ )</td>
<td>0.00077</td>
</tr>
<tr>
<td>c₃₄</td>
<td>( x₃y₄ )</td>
<td>0.00498</td>
</tr>
</tbody>
</table>
acres in the county would increase by its mean value, 8,050 acres. For sites (y_2), a one-unit increase implies that the number of sites would increase by 7.94. For fishing acres (y_4), a one-unit increase implies that the number of fishing acres would increase by 288. If we think of the price of income as equal to $1, then the denominator price, p_i, for each
shadow price ratio, equals the mean value of income reported in Table 1, $1.059 billion.

To illustrate how the shadow prices can be used to impute values for the conservation land attributes, consider the number of acres of conservation land in a county, \( y_3 \). The shadow price ratio is \( p_3 / p_1 = 0.017 \) for a county with mean outputs and mean inputs. A one-unit increase in \( y_3 \) would increase county conservation acres (other than acres available for fishing) by 8,050. What is the value of lost income that would support this increase? Given that one-unit of income equals the mean value of income, its price is \( p_1 = $1.059 \) billion. To calculate the shadow price of county conservation acres, \( p_3 \), take \( p_3 = p_1 \times 0.017 = $1.059 \) billion \times 0.017 = $18,003,000, or $2,236 per acre. Similar calculations can be performed for the number of conservation land sites, \( y_2 \), and for the number of acres available for fishing or other water-based activities, \( y_4 \).

In Table 4, we present the means and range of values for each conservation attribute evaluated using each county’s level of outputs and inputs. The values for sites, acres, and fish are for an increase in output equal to their mean value. For each price, \( p_{ma} = p_1 \times [\partial D^{-o}_m(\cdot) / \partial y_m] / [\partial D^{-o}_m(\cdot) / \partial y_1] \), where the price of one unit of income is \( p_1 = $1.059 \) billion. The results are presented on a per unit basis and compared for urban and rural counties.

Sites are worth about $788,000 each for all counties and average $274,000 for rural counties and $3.8 million for urban counties. An extra acre of land added to a site averaged $1,320, ranging from $495 in rural counties and rising to $6,132 in urban counties. An extra acre of water available for fishing (or other water-based activities) is valued at $16,529, ranging from an average of $16,422 in rural counties to $17,149 in urban counties. Although the price per site and the price per acre of land were significantly different between urban and rural counties, the price per acre of water for fishing or other water-based activities was not. A possible explanation for the insignificant difference between rural and urban water acres is that congestion on urban lakes completely offsets the benefits of having lakes in close proximity.

Recall that the Missouri Department of Conservation spent more than 5 million dollars in 1994 to acquire 13,000 acres of forestland. These figures suggest a minimum price per acre of $385, with almost
all forestland located in rural counties. Given that the department then constructs parking, picnic areas, trails, and exhibits at various sites while using labor and capital for oversight and safety, our figure of $495 per acre in rural counties seems reasonable.

It is more difficult to find comparable figures for our estimate of $16,529 per acre of water at conservation sites. One possible comparison, however, is for a proposed lake that was never constructed. In the late 1980s, Cape Girardeau county and Bollinger county in southeast Missouri proposed building a recreational reservoir financed by a 1% bicounty sales tax. A feasibility report projected costs of land acquisition, dam and spillway construction, and county maintenance costs of $75.16 million for a 7,680-acre lake (Lemons 1989). On a per-acre basis, the projected cost was $9,786 in 1989. Inflating by the GDP implicit price deflator, the projected cost in 1994 dollars would have been $11,547. Citizens in the two counties subsequently rejected the tax levy, partially based on a public perception that the projected costs were too low. Based on this, our 1994 figure of $16,529 per water acre also seems to be a reasonable estimate.6

5. CONCLUSION

Growing concern among the public and policy makers about the costs of urban sprawl and the benefits associated with maintaining large tracts of undeveloped land necessitates the imputation of shadow values so that tradeoffs can be accurately assessed. In this article, we used the directional output distance function, which characterizes the production technology, to obtain shadow prices of the characteristics associated with Missouri conservation areas. To implement the model, we assumed that Missouri counties use labor, capital, and land to produce money income and conservation areas. A quadratic functional form was estimated using the linear programming least-absolute deviations approach to obtain estimates of the value of conservation sites, acres of land, and acres of water at Missouri conservation areas. The empirical results appear to be in line with other estimates and indicate that urban counties place higher values on additional sites and acres of land than rural counties.
Conservation sales and use tax funds’ managers at the Missouri Department of Conservation should find the results of our method useful in determining where to spend new tax revenues. Given two alternatives, sites equal in their characteristics except for county location, allocative efficiency would imply spending the money in the county that has the highest shadow value for the new site relative to the costs of acquisition. As the state inventory of conservation lands changes over time and as county employment patterns, capital use, and income generation change, the method can also be easily updated to reflect the new production technology and associated shadow prices.

NOTES

1. Strong or free disposability of outputs means that if \( y \in P(x) \), then for \( y' \leq y, y' \in P(x) \). Letting \( L(y) = \{ x : x \text{ can produce } y \} \), then strong or free disposability of inputs means that if \( x \in L(y) \), then for \( x' \geq x, x' \in L(y) \).

2. A second shadow price model can be similarly derived via Shephard’s (1970) output distance function. The output distance function is \( D_o(x,y) = \inf \{ \theta : (y/\theta) \in P(x) \} \) and \( D_o(x,y) \leq 1 \) if and only if \( y \in P(x) \). The output distance function can be recovered from the revenue function as \( D_r(x,y) = \sup_y \{ py/R(x,p) \} \) (Färe and Primont 1995). Applying the envelope theorem yields \( \nabla_y D_r(x,y) = R(x,p) \) and for two outputs, \( m \) and \( m' \), \( p_m = p_m' \left( \frac{\partial D_r(x,y)}{\partial y_m} \right) / \left( \frac{\partial D_r(x,y)}{\partial y_m'} \right) \).

3. These restrictions are a consequence of Equation 9.

4. We also estimated a translog output distance function and used the results described in Note 2 to calculate the shadow prices. The shadow price ratios for the translog function are \( p_2/p_1 = .017, p_3/p_1 = .010, \) and \( p_4/p_1 = .027 \). Further results on the translog parameter estimates, shadow prices, and efficiency levels are available from the authors upon request. English et al. (1993) provide details on the estimation procedure.

5. The LP problem used to estimate the directional output distance function is \( D_{n-1}(x,y;1) = \max \{ B : y + \beta i \leq Yz, x^k \geq Xz, z \geq 0 \} \), where \( i \) is an \( m \times 1 \) vector of ones. Using the same deflated outputs and inputs, \( x^k \) and \( y^k \), the mean estimate \( D_{n-1}(x,y;1) = 0.065 \) (s = 0.11).

6. The 1994 estimate for Bollinger county is $9,398 per water acre, and for Cape Girardeau county, it is $18,332 per water acre.

REFERENCES


Missouri State Board of Education. 1996. 1994-95 report of the public schools of Missouri. Missouri State Board of Education.


Renken, Tim. 1976. Vote puts state first in per capita conservation outlays. St. Louis Post-Dispatch, 4 November, p. 6A.


Rolf Färe is a professor in the Department of Economics and the Department of Agricultural and Resource Economics at Oregon State University. He has been interested in efficiency and productivity issues since studying with Ronald W. Shephard in the 1970s.
Shawna Grosskopf is a professor in the Department of Economics at Oregon State University. She received her Ph.D. in 1977 from Syracuse University. Her research interests include public sector economics and performance measurement.

William L. Weber is a professor in the Department of Economics at Southeast Missouri State University. His research interests include valuing public goods and measuring efficiency and productivity change when firms produce polluting outputs.
Factors of production that cannot be moved from one activity to another due to their intrinsic nature, location preferences, mobility restrictions, or licensing requirements were featured prominently in the tax literature of the 1970s. Immobile factors, however, often produce inputs for other sectors. Several examples of this type that enhance and enrich some well-known existing models are presented. The value-adding process and cross-sector connections are explicitly specified. The new tax-incidence results often resemble those in mobile-factors-only (mfo) models in spite of one or more sector-specific inputs. Numerical examples, based on stylized U.S. data, illustrate the results and highlight the difficulties that arise in defining equivalent specifications. Goods mobility offsets some effects of factor immobility, but the computed tax elasticities are rarely the same as in mfo models. The Marshallian short-run/long-run distinction, blurred somewhat by production linkages, does not disappear.

SPECIFIC INPUTS, VALUE-ADDED, AND PRODUCTION LINKAGES IN TAX-INCIDENCE THEORY

KUL B. BHATIA
University of Western Ontario

1. INTRODUCTION

In an economy with two sectors, each producing a final good with the help of a mobile factor (L) and a sector-specific input (K), a Harberger-type neoclassical model leads to the well-known tax-incidence proposition that a partial output tax, or a tax on L in one industry, will hurt the mobile factor throughout the economy and benefit K in the untaxed sector. This result, formally derived by McLure (1971) in a $3 \times 2$ final-goods-only (fgo) model is typical of a class of models that deal with topics such as factor immobility, regional incidence of taxes, and short-run and long-run considerations (Bhatia 1989).

AUTHOR’S NOTE: Thanks are due to Professor J. Ronnie Davis for his patient handling of this article and to an anonymous referee whose comments led to a substantial reworking of the analytical derivations.
Such models generally assume primary factors of production (undeveloped land, unskilled labor) that produce only final goods, and the immobile inputs are not assigned any role outside their specific domains. Although these assumptions are appropriate in many cases (licensing requirements, mobility restrictions, location preferences) and have been useful in many fields of economics (Jones 1971b), the increasingly complex structures of modern economies present many situations in which primary factors inevitably go through some value-adding process (excavation, packaging, transportation) before they can be used for producing final goods, and sector-specific inputs are linked with production activities in other sectors. Examples range from simple, conventional activities to sophisticated, cutting-edge technologies. For instance, among conventional activities are latex, iron ore, and hybrid corn, all raw materials gathered, mined, or grown in the primary sector for processing or production elsewhere in the economy, and among sophisticated technologies, one may include computer programmers in Silicon Valley who create special effects for movies as well as customized computer-assisted designs (CADs) or templates for computer-assisted manufacturing (CAM). Additional examples can be found in software and hardware for telecommunications, genetic-engineering products, and related new technologies. Most specific inputs in these categories will have a definite value-added component, and they are likely to be used outside their sector of origin.

The main objective of this article is to consider tax-incidence questions in the presence of produced specific inputs (p.s.i.s). Their contribution to all sectors of the economy and the value-adding process are explicitly modeled while retaining the key feature of the McLure formulation, namely, a primary factor that does not move between the two final-good activities. Simply replacing one such immobile factor by a specific input produced in the other sector is enough to dislodge the tax-incidence result noted above: A selective production tax, rather than hurting the mobile factor, may actually benefit it, and the specific factor in the untaxed sector may lose. Moreover, a partial tax on the mobile factor, which mimics an output tax in the McLure model (subject to some exceptions), may have very different effects in this framework. In spite of the sector-specific inputs, the tax-incidence
propositions are often similar to those in a typical mobile-factors-only (mfo) model.1

Given the popularity of large-scale computable general equilibrium (cge) models, the analytical approach and the small models used in this article would call for an explanation if not an outright defense. During the past three decades, notable advances in taxation theory have been made with Harberger-type models—including the McLure formulation—which arguably also provide the nucleus for many cge tax models. Although the cge apparatus can incorporate all types of inputs, at least in principle, it is better suited to answering specific questions from a given data set than to deriving general results or explaining why and how particular outcomes come about. The latter tasks are better performed in suitable, analytically tractable models, even though they may lack the detail and the large dimension of a typical cge set-up. At any rate, as far as we can tell, p.s.i.s per se have not been featured in any empirical work in taxation.

Apart from questions of tax incidence, two related issues that arise in many areas of economic analysis and policy will also be taken up: First, almost all the new results emerge when one sector produces an input for another, as in the CAD/CAM example. Can such cross-sector connections offset or eliminate the effects of factor immobility? Second, the Marshallian short-run/long-run distinction based on factor mobility, which has a long and distinguished history, suggests that all inputs may be regarded as mobile in the long run, although some of them may not be so in the short run. If immobile factors of production do in fact produce mobile inputs, is the short run still a distinct and meaningful concept? Such questions can be answered by setting up comparable models, but which models may be regarded as equivalent is often a complex issue involving both theoretical and empirical considerations. One contribution of this article is to show that alternative specifications that may be viewed as analytically equivalent will not always generate the same outcomes in a plausible empirical setting.

The approach adopted here is a blend of theory and computational techniques, drawing on the substantial body of work in both areas. Many of the issues considered in the theoretical sections of the article have a strong empirical dimension. The analytical results about tax incidence or model equivalence, for instance, seldom go beyond indi-
cating the direction in which the rental-wage ratio will move, and even that depends in many cases on actual parameter values and other empirical magnitudes. This is where cge illustrations—based on a stylized U.S. data matrix in Solow (1986)—will be helpful, especially in providing plausible examples of outcomes that appear to be ambiguous in the most general case.

The main analytical framework is laid out in the next section, and questions of tax incidence in that setting are considered in Section 3. The results are compared with those derived from mobile-factor models in Section 4, where the issue of model equivalence is also discussed. Section 5 is devoted to numerical illustrations, and the conclusions are summarized in Section 6.

2. THE ANALYTICAL FRAMEWORK

The principal model can be introduced by formalizing a simple, manufacturing/agriculture example. Let the economy produce two final goods: rubber products (X₁) and food (X₂), the latter in the agricultural sector, which also supplies latex (X₃) to be used specifically in manufacturing X₁. Land (K) is the specific input in agriculture, and labor (L), the only mobile factor, is employed in all three activities. X₃ is the direct result of the value added by labor in a section of the agricultural sector. This specification will apply to a technology setting as well if agriculture is replaced by a software sector that employs specialized programmers (K) to produce computer games for home entertainment (X₂) and CADs (X₃) for use in the production of another final good (X₁).

Among other assumptions, the two primary factors have exogenous endowments (L and K); all markets are competitive, with no excess demand or supply anywhere; and the three production functions are linear homogeneous. Also, consumers (with identical, homothetic preferences) provide K and L, there are no taxes in the initial equilibrium, and when taxes are levied, the tax revenue is rebated to the taxpayers in a lump-sum fashion. These are standard assumptions in the tax literature that provides the context for the present framework, so they will facilitate comparisons with existing results. Denoting input-
output coefficients by $a_{ij}$, the full employment conditions can be written as

$$a_{l1}X_1 + a_{l2}X_2 + a_{l3}X_3 = L$$

(1)

$$a_{k2}X_2 + a_{k3}X_3 = K$$

(2)

$$a_{31}X_1 = X_3.$$  (3)

Combining Equation 3 with the other two, and defining $R_{l1} = a_{l1} + a_{31}a_{l3}$, we get

$$R_{l1}X_1 + a_{l2}X_2 = L$$

(4)

$$a_{k2}a_{31}X_1 + a_{k2}X_2 = K.$$  (5)

Here, $R_{l1}$ represents total usage of labor by $X_1$, direct as well as through $X_3$, and because of this linkage, farmland also becomes an indirect input for $X_1$ although it is directly used only in agriculture where it is physically located.

2.1. COMPARISON WITH EARLIER MODELS

This framework, although it may be regarded as having two sectors and two mobile factors (labor is physically mobile, and farmland effectively so, through $X_3$), is different from the original Harberger model, which has only primary factors of production and final goods. Because of the intermediate good, $X_3$, some overlap with the analytical models that incorporate intersector linkages is inevitable, but there is no formal treatment of sector-specific inputs of this type in that literature, so the most one can expect from them are a few useful concepts and solution techniques.

Because farmland is physically immobile between the two final-good sectors, this specification is closest to the McLure set-up. It has the same dimension, $3 \times 2$, and although the number of factors and goods is reversed (two primary factors and three goods here), the same words can still describe both models: two sector-specific inputs, one mobile factor of production, and two final goods. Formally, one of McLure’s primary immobile factors of production has been replaced
by a p.s.i. produced in the other sector, and it has the remarkable side effect that relative factor intensities are comparable across activities, which cannot be meaningfully done in the McLure model but is a hallmark of mfo models. Interestingly, if X₂ were to be used by X₁ instead of X₁, labor will be the only input common to the two final goods, so their factor intensities will not be comparable. This complication, however, will not arise in our examples if latex production shifts to the manufacturing sector, or some software specialists move there because X₁ and X₂ could still be ranked by their L-K ratios. Both primary factors, moreover, will be mobile between the two final goods, and the present analysis, with some modifications, will continue to apply.

On the whole, considering these similarities and differences, one should expect a mixture of results: some similar to those in mfo models, others closer to the tax-incidence propositions in the 3 × 2 fgo specifications, and a few hybrids too.

2.2. SOLVING THE MODEL

Following Jones (1965), the solution procedure entails total differentiation of the full employment conditions and other equations while incorporating the perfectly competitive assumptions (zero profits, factor rewards equal to value of marginal product, and so on). Let us define λᵢⱼ as the proportion of the ith input used directly or indirectly by the jth activity. Thus, λₖ₂ = K₂/K and λₙ₁ = (L₁ + L₂)/L. Also, (λₙ₁ + λₖ₂) = (λₙ₁ + λₖ₂) = 1. If asterisks indicate proportional changes, total differentiation of Equations 4 and 5 leads to

\[ \lambda_{L1}X_1^* + \lambda_{L2}X_2^* = -\lambda_{L1}R_{L1}^* - \lambda_{L2}a_{L2}^* \]

\[ \lambda_{K1}X_1^* + \lambda_{K2}X_2^* = -\lambda_{K2}a_{K2}^* - \lambda_{K1}(a_{K1}^* + a_{L1}^*) \]

Turning to factor incomes, let ps denote direct input shares whereas θs are “total” shares, reflecting both direct and indirect usage. With w as the wage rate, r the return to K, and p₁ the unit price of X₁, ρ₁₂ = (wL₁/p₂X₂), θ₁₁ = ρ₁₁ + ρ₁₂θ₁₂, and other ρs and θs can be analogously defined. To minimize unit costs, firms base their input choices on relative input prices, so R_{L1}^* and the aᵢᵢs can be solved in terms of input prices.
and elasticities of substitution ($\sigma$). For instance, $a^*_j = -\rho_{il}K_3(w^* - r^*)\sigma^i$ and $R^*_j = -(w^* - r^*)\rho_{ij}K_3(\rho_{il}K_3\sigma^i + \rho_{il}\sigma^i)$, the latter reflecting substitution possibilities in p.s.i. production as well as in the using sector ($X_j$).

The zero-profit conditions in the final goods markets can be written as

$$\theta_{L1}w^* + \theta_{K1}r^* = p_1^*, \ (8)$$

and

$$\rho_{L2}w^* + \rho_{K2}r^* = p_2^*, \ (9)$$

where the presence of $\theta$s indicates that Equation 8 already incorporates the zero-profit condition for $X_3$. Following Atkinson and Stiglitz (1980, Lecture 6), the demand side of the model is summarized by $(X_1^* - X_2^*) = \sigma_0(p_2^* - p_1^*)$, where $\sigma_0$ (defined to be positive) is the elasticity of substitution in demand.

### 3. TAX INCIDENCE

There are no taxes in any of the equations specified thus far. Before introducing them, it is worth noting that tax incidence in models of this type depends on what happens to the relative income shares of the primary factors of production, that is, on changes in input prices because of the full-employment assumption. The model, therefore, is solved for $(r^* - w^*)$ for which a two-step process seems convenient: Determine $(X_1^* - X_2^*)$ from the supply-side equations after incorporating the relevant tax terms, and then equate it to the corresponding change in demand. Choosing $w$ as the numeraire (so that $w^* = 0$) and setting all initial prices to unity will simplify some of the derivations. We shall consider a selective output tax and a partial tax on labor, both “small,” in keeping with the requirements of the analytical solution process, and the ratio $r^*/t^*$ will be the elasticity of the rental-wage ratio with respect to a given tax. The selected taxes provide an interesting contrast with the results in the McLure model, and many other taxes can be an-
alyzed in an analogous manner. The food/latex example mentioned at the beginning of Section 2 will be the backdrop for the rest of this section.

3.1. SELECTIVE OUTPUT TAXES

When a tax on the output of \( X_1(T_1) \) is levied, Equation 8 can be written as \( \theta_L w^* + \theta_K r^* + T_1^* = p_1^* \); proportional change in relative demand, after substituting for \( p_1^* \) and setting \( w^* = 0 \), is then given by \( X_1^* = \sigma_D(\rho_{K2} - \theta_{K1})r^* - \sigma_D T_1^* \), and the corresponding change in relative supply follows from Equations 6 and 7:

\[
|\lambda|(X_1^* - X_2^*) = [\rho_{K3}(M_1 - S_1)\sigma^1 + (M_2 - S_2)\sigma^2 + (M_3 - S_3)\sigma^3]r^*,
\]

where the Ms and Ss are positive, being sums or products of \( \lambda s, \rho s, \theta s \) (all positive fractions), and \( \sigma s \) are defined to be nonnegative. For instance, \( M_1 = \lambda_L \lambda_K \lambda_{L1}/\theta_{K1}, \) and \( S_1 = (\lambda_{L2} \lambda_{K1} + \lambda_{K2} \lambda_{L1} \rho_{K1}/\theta_{L1})\rho_{L1}. \) The determinant, \( |\lambda| \), reflects the factor intensities of the two final goods, \( X_1 \) and \( X_2, \) based on total usage of each input. Thus, if \( X_1 \) is relatively labor intensive, after taking into account the labor employed in producing \( X_1, \) \( |\lambda| > 0. \)

At this stage, it is useful to recognize a local stability condition along the lines of Atkinson and Stiglitz (1980, chap. 6), which postulates that a fall in the wage-rental ratio \( r^* > 0 \) should lead to a decline in the relative price of the labor-intensive good and also its output. In other words, the “supply curve” of \( X_1/X_2 \) is upward sloping. Here, it implies that Equation 10 and the expression within brackets on its right-hand side will be negative; that is, if \( r^* > 0 \) and \( |\lambda| > 0, \) \( (X_1^* - X_2^*) < 0. \) This condition will be invoked at several points in the analysis that follows.

Equation 10 and the demand function yield

\[
r^* = \sigma_D |\lambda| T_1^*/D_1,
\]

where \( D_1 = \sigma_D(\rho_{K2} - \theta_{K1})|\lambda| - [\rho_{K3}(M_1 - S_1)\sigma^1 + (M_2 - S_2)\sigma^2 + (M_3 - S_3)\sigma^3]. \) The stability condition just discussed dictates that the terms within the square brackets are negative, and the first term in \( D_1 \) will be
positive because $\sigma_0 > 0$ and $|\lambda|$ and $(\rho_{k3} - \theta_{k1})$ have the same sign (both positive, e.g., when $X_1$ is relatively labor-intensive). $D_1$, therefore, will be generally positive, and the sign of $r^*$ will depend on the numerator of Equation 11.

Result 1: The incidence of a selective output tax depends on relative factor intensities; the primary specific input ($K$) benefits ($r^* > 0$) only if the taxed industry is relatively intensive in the nonspecific factor ($L$).

The tax leads to a reduction in the output of $X_1$, thereby reducing the demand for labor as well as for its specific input, $X_3$. If the taxed industry is relatively labor intensive, there will be an excess supply of labor in the economy, and the wage-rental ratio will fall to restore full employment.

A remarkable aspect of this straightforward result is that it be-speaks no sector-specific inputs, produced or primary; it rather reads like a proposition from tax models with all mobile factors of production. The similarity is not fortuitous because as noted earlier, in connection with the full-employment conditions, both inputs, in effect, are mobile. Even though farmland cannot be physically moved to the manufacturing sector, land-labor ratios of the two final goods can be compared, and they are sufficient to determine the direction of change in the wage-rental ratio as in the conventional $2 \times 2$ framework with all mobile inputs.

In the $3 \times 2$ framework, this tax always benefits the primary specific input in the nontaxed activity simply because its land-labor ratio declines due to an influx of labor from $X_1$. The adjustment process is complicated now by $X_3$ because although some labor is released from $X_1$, there might well be an excess demand for labor overall, so $r^*$ could be negative.

An analogous reasoning applies to a partial output tax on $X_2$ (in this case, $r^* = -\sigma_0|\lambda|T_1^*/D_1$). When the taxed industry, $X_2$, is relatively labor intensive, $|\lambda| < 0$, and $r^* > 0$. On account of the tax, the output of $X_2$ declines, while that of $X_1$ increases, bringing in its wake a greater demand for $X_3$, eventually leading to a higher rental for land. There is nothing comparable to this outcome in the $3 \times 2$ framework, where a specific factor cannot possibly benefit from a tax on the output of a good it helps directly produce.
3.2. PARTIAL FACTOR TAXES

A factor tax induces cost-minimizing firms to substitute away from the taxed input, and their ability to do so depends on the relevant elasticity of substitution. For a tax on labor directly employed in X₁, therefore, σ₁ will be of special interest. For an output tax on X₁, this elasticity affected the magnitude of r*, as a component of D₁ in Equation 11. Here, it can also change the sign of r* and lead to a situation in which, unlike Result 1, r* > 0 even when the taxed industry is relatively land intensive. A partial tax on labor in X₂ also has some interesting implications.

3.2.1. A tax on L₁(tL₁)

Note, first, that this tax will add a new term, ρL₁tL₁, to the left-hand side of Equation 8, and all the expressions dealing with input choices in X₁—a₃₁, a₄₁, and R₁₄—will be affected. For instance, a₃₁ will have an extra term, ρL₁σ₁tL₁. Once again, from Equations 6 and 7, after plugging in the modified aₙₛ’s, the proportional change in the ratio of final outputs is

$$|\lambda|(X₁' − X₂') = |\rho_{KL}(M₁ − S₁)\sigma₁ + (M₂ − S₂)\sigma₂ + (M₃ − S₃)\sigma₃|r^* + (S₁ − M₁)\sigma₁tL₁.$$  (12)

As in Section 3.1, equating proportional changes in demand and supply yields the solution for r*:

$$r^* = |\sigma₁\rho_{KL}| |\lambda| + (S₁ − M₁)\sigma₁tL₁/D₁.$$  (13)

Earnings of labor directly employed in X₁ provide the base for this tax, and that is why ρL₁, its input share, is in the numerator of Equation 13. As discussed earlier, D₁ is positive; the sign of r*, therefore, depends on the |λ| and σ₁ terms. What the tax does is push up the unit cost of producing X₁, its output falls, and demand for its inputs is affected, as it did in the wake of the output tax considered earlier. The principal new element is the appearance of σ₁ in the numerator of r*, and it will have a noticeable effect on the incidence of this tax.
Result 2: If the taxed industry is relatively labor intensive, labor throughout the economy will suffer from a partial tax upon itself. When this result holds, \( r^* > 0 \), and it can be explained by noting that because \( \sigma^i \geq 0 \) by definition, and \( (S_i - M_i) > 0 \) due to the stability condition noted earlier, \( |\lambda| > 0 \) will be sufficient to make \( r^* \) positive. The tax will lead to an excess supply of labor, and the wage-rental ratio must fall to restore full employment. In this regard, this tax works very much like the partial tax on the output of this industry, and \( \sigma^i \) actually reinforces the effect of relative factor intensities.

Result 3: Labor can benefit from a partial tax on itself only if the taxed industry is relatively land intensive.

For this outcome, \( r^* \) must be negative, and \( |\lambda| < 0 \) provides a necessary condition for it. Large values of \( \sigma^i \), however, can still make \( r^* \) positive. Labor normally benefits if the taxed industry is relatively land intensive because other things being equal, as \( X_1 \) contracts, a smaller excess supply of labor will ensue, but not necessarily so if \( X_3 \) can be substituted for labor with considerable ease. In the present case, from labor’s standpoint, values of \( \sigma^i \) larger than \( \sigma_1 \rho_1 |\lambda|/(S_1 - M_1) \) will outweigh any beneficial effect of a favorable configuration of land-labor ratios. For an output tax, a “high” value of \( \sigma^i \) can at most force \( r^* \) to zero (because it appears only in the denominator of Equation 11), without ever reversing its sign. In this regard, the p.s.i. model parts company with the McLure formulation in which \( t_{L1} \) and \( T_1 \) generally end up with the same, positive sign for \( r^* \). Here, Result 1 shows that \( r^* \) can be negative, and the other propositions indicate that the rental-wage ratio may move in opposite directions for the two taxes.

Result 4: If the taxed industry uses labor and the intermediate input in a fixed ratio, the wage-rental ratio will move in the same direction whether the tax is on output or labor.

Fixed input-output coefficients rule out the possibility just noted of divergent outcomes for \( t_{L1} \) and \( T_1 \). When \( \sigma^i = 0 \), Equations 13 and 11 will have the same sign, determined by \( |\lambda| \). The adjustment process de-
scribed in connection with Result 3 is constricted, and relative factor intensities alone matter.

3.2.2. A tax on $L_2(t_{L2})$

The solution process is symmetrical to the one just followed for $t_{L1}$, and all the equations pertaining to $X_2$ will be modified. Parallel to Equation 13, we get

$$ r^* = \left[ -\sigma_{L2} \rho_{L2} |\lambda| + (S_2 - M_2) \sigma^2 \right] t_{L2}^* / D_1. $$

(X_2 is now the taxed industry, and if it is relatively labor intensive, $|\lambda| < 0$, and $(S_2 - M_2) > 0$ because of the stability condition noted earlier. Results 2, 3, and 4, therefore, once again hold. Note, in particular, that the elasticity of input substitution will offset the effects of a relatively land-intensive taxed industry in this case ($|\lambda| > 0$), so $r^*$ can be positive or negative. The computations reported in Section 5 will illustrate both possibilities.

4. COMPARISONS WITH MOBILE FACTOR MODELS

The analytical propositions derived thus far on the whole resemble the results in mobile-factor models, where the incidence of partial factor taxes does depend on factor intensities and $\sigma$s, as in the present set-up. The closest model of this type is the two-sector Harberger model with both L and K mobile. In fact, McLure (1974) described his formulation as “the Harberger model with one immobile factor.” The Harberger framework and its extensions that include intermediate inputs (e.g., Bhatia 1982) will be helpful in answering the two questions raised in the Introduction: Can goods mobility offset the effects of factor immobility, and is the Marshallian short run still a useful concept? The challenge is to find alternative specifications that may be regarded as comparable and equivalent. Empirical considerations will also be important in this task because $\lambda$s, $\rho$s and $\theta$s, which figure prominently in the expressions for $r^*$, are largely determined by the numbers, functional forms, and the elasticities selected in a given situation. Analyti-
cal issues are taken up first, and questions of a more empirical nature will be deferred to the next section.

4.1. GOODS MOBILITY AND FACTOR IMMOBILITY

The goods-mobile production function implied by the p.s.i. model in Section 2 can be written as \( X_1 = f(L_1, g(K_3, L_3)) \), where \( g() \) is the production technology for \( X_3 \). This nested structure implies that were \( K_3 \) and \( L_3 \) to be directly employed in \( X_1 \) instead of being packaged in \( X_3 \), \( \sigma_{13} \) would be equal to \( \sigma_{L1L3} \) and \( \sigma_{L1K3} \). A comparable mfo production function then suggests itself, namely, \( X_1 = h(L_1, L_3, K_3) \), a three-input specification with partial elasticities of substitution (Allen 1967), one for each pair of inputs. These functional forms imply that when the two partial elasticities, \( \sigma_{L1L3} \) and \( \sigma_{L1K3} \) are equal, \( f() \) and \( h() \) may be regarded as equivalent. This technical point can be restated as a lemma.

*Lemma 1:* If labor can be substituted with equal ease for all inputs in the mfo specification, the mfo and p.s.i. production functions will be equivalent.

Logically, this lemma suggests a general result as well as an empirical test.

*Result 5:* If tax-incidence results are not sensitive to the elasticity restrictions in Lemma 1, goods mobility will offset the effects of immobile factors of production.

Pursuing these production functions further, note that when labor is homogeneous, \( L_1 \) and \( L_3 \) can be added together, and if land usage can also be shifted between the two final goods, \( X_1 \) gets a simple two-input production function like the ones in Section 2, except that \( K_3 \) replaces \( X_3 \). The full employment and zero-profit conditions then need to be modified accordingly, and \( r^* \) replaces \( p^*_3 \) in the definition of \( \sigma^* \). Anticipating the discussion in Section 4.2, the resulting equations will also characterize a long run in the McLure specification (Bhatia 1989), and the \( 2 \times 2 \) mfo model in Harberger (1962) in fact. The corresponding expressions for \( r^* \) (Mieszkowski 1967) are very similar to Equations 11 and 13, which suggests that goods mobility would indeed offset the
effects of factor immobility. Whether the offset is partial or complete is an empirical matter to which we shall turn in Section 5.

Two qualifications that may lead to different incidence results, nonetheless, are worth noting: First, when all inputs are mobile and used directly in producing the two final goods, $|\lambda|$ may have a different sign than in the produced-input specification, a type of factor-intensity reversal. In other words, when $X_3$ is used by $X_1$, its land-labor ratio is, say, smaller than in $X_2$, but factor intensities get reversed when all inputs are mobile and used directly for producing $X_1$ and $X_2$. The rental-wage ratio, therefore, may move in opposite directions for the same tax in the two formulations. Second, when $t_{i_s}$ is considered, $\sigma^1$ is the elasticity of substitution between labor and land in the sfo case, rather than between labor and $X_3$. Although both are defined to be nonnegative, $\sigma^1_{L,K}$ and $\sigma^1_{L,3}$ need not be equal in all cases. They may be premultiplied by different terms anyhow in the solutions for $\tau^*$, so the same data set may lead to different results (one such outcome will be highlighted in Section 5.2).

4.2. SHORT RUN VERSUS THE LONG RUN

The Marshallian long run, strictly defined as the absence of immobile inputs, will occur when $K$ actually becomes mobile. One may thus think of a $2 \times 3$ framework: two mobile primary inputs and three goods, only two of which are needed for final demand, the third treated as an intermediate input for $X_i$. The production function for $X_i$ then becomes $X_i = f(K_i, L_i, X_3)$, and $R_{K_i} = a_{K_i} + a_{K_3}a_{3i}$, to reflect total usage of land, analogous to $R_{L_i}$ for labor. The full employment condition for land can be written as $R_{K_i}X_i + a_{K_3}X_3 = K_i$, and $\theta_{K_i}$ in the zero-profit condition (Equation 8) is computed from $R_{K_i}$. Labor and land thus are treated alike. The expanded production function will introduce some new (partial) elasticities of substitution, $\sigma^1_{L,K}$ and $\sigma^1_{L,3}$, to reflect new margins of substitution, and there is the possibility of complementary inputs ($\sigma < 0$), which does not arise in a two-input production function. There is, of course, the chance of a factor-intensity reversal also; that is, $R_{i,s}$ may rank $X_i$ and $X_3$ differently than the $a_{i,s}$.

For these reasons, it seems likely that the incidence results derived from this formulation will not be the same as in the p.s.i. specification.
From a technical standpoint, the three-input functions are not comparable with the two-input production functions deployed thus far; therefore, instead of detailed comparisons, the long-run specification is briefly explored to derive one or two results to be set against the “short run” results in Section 3. The model can still be solved for $r^*$, and we shall focus on $t_{L1}$ because a partial output tax will simply reiterate the importance of relative factor intensities.

### 4.2.1. Partial tax on labor, long-run incidence.

With $t_{L1}$ in place, cost-minimizing firms will try to substitute $K_1$ and $X_3$ for labor if they can. Although some steps in the derivation become more complex because of the extra elasticities of substitution, the solution process remains intact, and

$$r^* = \frac{[A \sigma_D + (\gamma K_2 L_2 + K_1 L_1 (\theta_{K1} - \theta_{L1})) / \theta_{K1} \theta_{L1}] (\rho_{K1} \sigma_{LK} + \rho_{L1} \rho_{K1} \sigma_{LK}) \rho_{L1} t_{L1}}{D_2}.$$  \hspace{1cm} (15)

where $A$, defined as $(L_1/L_2 - K_1/K_2)$, incorporates the direct and indirect usage of all inputs, $\gamma = (\theta_{L1} \lambda_{K1}/\lambda_{K2} + \theta_{K1} \lambda_{L1}/\lambda_{L2})$, and $D_2$ is very similar to the denominators in the $r^*$ expressions derived earlier. Every term in $\gamma$ is positive, and for reasons analogous to those noted in connection with $D_1$, $D_2$ also will be positive; therefore, the sign of $r^*$ once again will be determined by the terms in the numerator. Compared to Equation 13, there is an additional term in the numerator of $r^*$ that reflects possibilities of directly substituting labor for the now mobile $K$. Some long-run results readily follow from the $r^*$ solution in Equation 15.

**Result 6:** So long as the taxed industry is relatively labor intensive, and there are no complementary inputs, land owners will benefit from a partial tax on labor in $X_1$.

In this situation, $A$, $\sigma_{Lk}^1$, and $\sigma_{L3}^1$ are all positive, $\theta_{L1} > \theta_{K1}$, so $r^* > 0$. The tax will definitely cause some labor to be released from $X_1$, but a “small” elasticity of substitution in demand and/or similar input ratios for the final goods (A close to zero) will tend to limit the consequent excess supply of labor. Moreover, a negative $\sigma$ in the numerator of
Equation 15, indicating complementarity between say labor and \( X_3 \), may turn the tables on capital owners and lead to a negative \( r^* \).

**Result 7:** The larger is the elasticity of substitution between land and the intermediate good in the taxed industry, the greater will be the tendency for the wage-rental ratio to remain unchanged.

Land, now mobile across industries, is the untaxed primary factor of production. The relevant elasticity, \( \sigma_{K3} \), is an element of \( D_2 \); it does not appear in the numerator of Equation 15, therefore, it cannot affect the sign of \( r^* \). In the limit, as \( \sigma_{K3} \to \infty \), \( r^* \to 0 \), which implies that labor and land will bear the burden of the tax in proportion to their initial factor shares.

These long-run outcomes could not happen in the model set out in Section 2, although they would seem to be a logical extension of the analysis presented there. Other results of this sort can be written down, by selecting extreme values of the elasticities (fixed proportions, “large” values of \( \sigma_{L3} \), and so on), but most of them will have strong empirical overtones. Considering additional theoretical alternatives, therefore, will have a rather limited payoff. As in Result 2, the factor-intensity term and the ones involving the elasticities of substitution must work at cross purposes to create any chance of switching the sign of \( r^* \). That implies either a taxed industry that uses land relatively intensively (Result 3) and/or some complementary inputs. The numerical examples in the next section will shed more light on some of these points.

All in all, summarizing the results presented thus far, it seems that p.s.i.s of various sorts would lead to tax incidence scenarios that differ considerably from those in McLure-type fgo specifications. Many results share some aspects of the outcomes in mobile-input models, but only rarely will the two coincide, and under rather severe restrictions on the parameters. Although cross-sector connections of the goods and services produced by immobile primary factors may offset the effects of factor immobility, the Marshallian short-run/long-run distinction will continue to matter.

The analytical propositions, even those with a touch of the long run, rarely go beyond indicating the direction in which the rental-wage ra-
tio will move (the sign of \( r^* \)), and it appears that the sign of \( r^* \) would be preserved, especially for output taxes, if factor-intensity ranks of the final-good industries, computed with due regard to direct and indirect usage of all inputs, remain unchanged under different specifications. Factor taxes generally lead to more complex situations, and it remains to be seen whether comparable and analytically equivalent formulations will generate the exact same empirical outcomes.

5. NUMERICAL ILLUSTRATIONS

The goal of this section is to get a sense of the importance of the many references to empirical matters throughout the theoretical sections of the article. For example, Result 3 indicates a possible negative sign for \( r^* \); there are the elasticity restrictions in Section 4.1, which may define comparable production functions; and the broader issue of identical empirical results from analytically equivalent models begs to be explored further numerically. Before venturing into this challenging and sometimes murky terrain, we shall try to illustrate the more definitive theoretical propositions to establish the credentials of the data set and the computational techniques.

Two options are open: The first, evident in Harberger (1962), Bhatia (1983), and Solow (1986), among others, is to calculate \( \rho_s \), \( \theta_s \), and input ratios for each industry from the data, find appropriate elasticity estimates, and plug all of them into the various analytical solutions for \( r^* \). Alternatively, following Shoven and Whalley (1972), actual functional forms (mostly CES production and utility functions) can be specified to calibrate the initial equilibrium, and a new general equilibrium then can be computed for each tax regime in a comparative-static exercise, much like the analytical derivations earlier in the article. For “small” tax changes in small-dimensional models, both approaches should yield similar results. We shall opt for the cge approach mainly because of available software that can readily compute a large number of comparable cases. Otherwise, one must resort to approximations, or perform cumbersome computations of factor shares and input ratios in pretax and posttax situations every time, as in Harberger (1962).
The data (1974, United States) are adapted from Solow (1986, Table 1), a 2×2 input-output table aggregated from the transaction matrix in Hudson, Jorgenson, and O’Connor (1978). Solow applied these numbers, following the first approach mentioned above for the most part, to examine the empirical importance of interindustry flows in the general equilibrium model developed by Bhatia (1982, 1983). Because there are no sector-specific inputs in that model, the data, rather than analytical propositions or Solow’s painstaking calculations, are relevant for the task at hand. There is no solid information, however, on the basis of which sector-specific inputs of any sort can be definitely identified in the data matrix and isolated in agriculture, manufacturing, or high-tech sectors along the lines of the examples formalized in the theoretical sections of this article. The computations presented below, therefore, can at best be regarded as illustrative. In defense of the data, however, it is worth noting that most of the aggregates at least are actual numbers for the U.S. economy, which reflect, albeit in a stylized way, some aspects of its production structure; and perhaps more important, the same data set anchors illustrations of several interrelated analytical results for a range of likely parameter values.

With these caveats out of the way, we follow standard cge methodology (Rutherford 1988) for calibrating the model and computing counterfactual equilibria for a series of “small” (1%) taxes. Proportional changes in the rental-wage ratio, which logically correspond to the analytical solutions for r* derived in the article, are calculated from the equilibrium values in pretax and posttax situations, and the

<table>
<thead>
<tr>
<th></th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1</td>
<td>870.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_2</td>
<td>437.89</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_3</td>
<td>270.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>600.42</td>
<td>129.65</td>
<td>72.33</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>870.75</td>
<td>437.89</td>
<td>270.33</td>
<td>1,308.64</td>
</tr>
</tbody>
</table>

NOTE: Xs are output levels; C is the consumption of each final good; and K and L, respectively, denote capital and labor employed in each industry.
numbers in Tables 2 through 5 may be called *elasticities of the rental-wage ratio with respect to a given tax*, or simply *tax elasticities*.

### 5.1. THE BENCHMARK DATA

The basic data for the initial equilibrium in the p.s.i. formulation are presented in Table 1, where the primary specific factor, $K$, is allocated between $X_2$ and $X_3$, roughly in the ratio of 3:2.\(^6\)

Taking into account total usage, $(L/K)$ is 3.4 and $(L/K) = 0.4$, so that $X_1$ is relatively labor intensive and $|\lambda| > 0$. McLure’s 3 $\times$ 2 specification can be approximated by assuming that a portion of $K$, actually $K_3$, in the p.s.i. formulation, is located in and specific to $X_3$. The $X_1$ column in Table 1, accordingly, replaces the row entry for $X_3$, by 198.0 for $K$, and labor employed in $X_1$ increases to 201.98. The rents, $r_1$ and $r_2$, will be determined as a residual in each industry. These numbers will also work for the 2 $\times$ 2 mobile factor case, except that the two

| TABLE 2: Effect of Taxes in $X_1$ on Wage-Rental Ratios |
|-----------------|-----------------|-----------------|
|                  | $r_1^*$         | $r_1^*$         |
|                  | $T_1$           | $T_1$           |
| p.s.i.           |                 |                 |
| (a) All $\sigma$'s = 1.0 | .0045 | .0075 |
| (b) $\sigma_{L3} = 0.5$; other $\sigma$'s = 1.0 | .0053 | .0063 |
| (c) $\sigma_{L3} = 0.3$; other $\sigma$'s = 1.0 | .0058 | .0057 |
| 2 $\times$ 2 mfo |                 |                 |
| (a) All $\sigma$'s = 1.0 | .0036 | .0075 |
| (b) $\sigma_{L3} = 0.5$; other $\sigma$'s = 1.0 | .0047 | .0062 |
| (c) $\sigma_{L3} = 0.3$; other $\sigma$'s = 1.0 | .0054 | .0067 |

**NOTE:** p.s.i. = produced specific input; mfo = mobile-factors-only; fgo = final-goods-only. The numbers correspond to $r^*$ in the analytical results derived in the article. Each tax is levied at the rate of 1% in $X_1 – T_1$ on the value of output, and $T_1$ on the earnings of labor directly employed in the industry. The model developed in the article is p.s.i., 2 $\times$ 2 mfo refers to the Harberger (1962) mobile-factor model, and the 3 $\times$ 2 fgo specification is from McLure (1971). The $\sigma$'s denote elasticities of substitution.
### TABLE 4: Effect of a Wage Tax on the Wage-Rental Ratio

<table>
<thead>
<tr>
<th>p.s.i.</th>
<th>$r^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0016</td>
</tr>
<tr>
<td>(b) $\sigma_{L3}^2 = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0002</td>
</tr>
<tr>
<td>(c) $\sigma_{L3}^2 = 0.3$; other $\sigma$'s = 1.0</td>
<td>-0.0001</td>
</tr>
<tr>
<td>2 × 2 mfo</td>
<td>$r_1^*$</td>
</tr>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0016</td>
</tr>
<tr>
<td>(b) $\sigma_{LK}^2 = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0011</td>
</tr>
<tr>
<td>(c) $\sigma_{LK}^2 = 0.3$; other $\sigma$'s = 1.0</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

NOTE: p.s.i. = produced specific input; mfo = mobile-factors-only; fgo = final-goods-only.

### TABLE 3: Effect of a Wage Tax in $X_2(t_{L2})$ on Wage-Rental Ratios

<table>
<thead>
<tr>
<th>p.s.i.</th>
<th>$r^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0016</td>
</tr>
<tr>
<td>(b) $\sigma_{L3}^2 = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0002</td>
</tr>
<tr>
<td>(c) $\sigma_{L3}^2 = 0.3$; other $\sigma$'s = 1.0</td>
<td>-0.0001</td>
</tr>
<tr>
<td>3 × 2 fgo</td>
<td>$r_1^*$</td>
</tr>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0016</td>
</tr>
<tr>
<td>(b) $\sigma_{LK}^2 = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0011</td>
</tr>
<tr>
<td>(c) $\sigma_{LK}^2 = 0.3$; other $\sigma$'s = 1.0</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

NOTE: The $r^*$ values correspond to the solutions derived in the article. The tax rate is 1% of the wage earnings in $X_2$. $\sigma$'s denote elasticities of substitution. $X_1$ and $X_2$ use the same quantity of land in the pretax equilibrium in each case.

a. In the initial equilibrium, $X_2$ employs 201.98 units of labor but only 129.65 in the p.s.i. specification. The tax rate has been adjusted to yield the same tax revenue in the two cases.

### TABLE 4: Effect of a Wage Tax on the Wage-Rental Ratio

<table>
<thead>
<tr>
<th>Production function $^a$: $X_1 = f(L_1, K_3)$</th>
<th>$r^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0079</td>
</tr>
<tr>
<td>(b) Taxed industry $\sigma = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0074</td>
</tr>
<tr>
<td>(c) Taxed industry $\sigma = 0.3$; other $\sigma$'s = 1.0</td>
<td>0.0071</td>
</tr>
<tr>
<td>Production function: $X_1 = f(L_1, L_3, K_3)$</td>
<td>$t_{L1}$</td>
</tr>
<tr>
<td>(a) All $\sigma$'s = 1.0</td>
<td>0.0083</td>
</tr>
<tr>
<td>(b) Taxed industry $\sigma = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0076</td>
</tr>
<tr>
<td>(c) Taxed industry $\sigma = 0.3$; other $\sigma$'s = 1.0</td>
<td>0.0071</td>
</tr>
<tr>
<td>(d) $\sigma_{L1L3}^2 = \sigma_{L1K3}^2 = 0.5$; other $\sigma$'s = 1.0</td>
<td>0.0025</td>
</tr>
<tr>
<td>(e) $\sigma_{L1L3}^2 = \sigma_{L1K3}^2 = 0.3$; other $\sigma$'s = 1.0</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

NOTE: The $r^*$ values correspond to the solutions derived in the article.

a. The initial land-labor ratio is the same as in the p.s.i. (produced specific input) specification in Table 3.
b. See Section 4.1 for a discussion of restrictions on $\sigma$'s.
rental rates will be constrained to equality and \( \sigma^1 \) will be the elasticity of substitution between \( L \) and \( K \) rather than labor and \( X_3 \). In each of these specifications, thus, the two final-good industries use the same amount of \( K \), directly or indirectly, in the initial equilibrium, whereas the labor input would differ in some cases.\(^7\) Therefore, while considering a tax on labor, the tax rates will be adjusted, where necessary, to ensure equal tax revenues.

5.2. TAXES IN \( X_1 \) AND \( X_2 \)

Table 2 illustrates the results for taxes in \( X_1 \), and the rental-wage ratio rises in every case. \( X_1 \) is relatively labor intensive, so the signs of \( r^* \)'s agree with the analytical results in the article for every specification. In the \( f(x) \) computations, the signs of \( r_1^* \) and \( r_2^* \) are also what the McLure model predicts, and as in the p.s.i. formulation, the primary factor specific to the untaxed sector always benefits, although the size of \( r^* \) is not the same in every case. Therefore, one can have confidence in the data and the computational procedures while approaching some of the analytically ambiguous situations identified in Sections 3 and 4.

The first candidate is Result 3, which suggests an uncertain incidence outcome for a partial tax on labor in a relatively \( K \)-intensive industry. In this data set, \( X_2 \) is that industry and \( t_{x2} \) is such a tax. Recall that in this case, 

\[
\hat{r}^* = \left[ -\sigma \rho \hat{\lambda} \right] + (S_2 - M_2) \sigma^2 \left[ t_{x2} / D_1 \right] \quad \text{(Equation 14)}
\]

which will be negative (because \( |\hat{\lambda}| > 0 \) in this data set) except for “large” values of \( \sigma^2 \).

<table>
<thead>
<tr>
<th>TABLE 5: Effect of a Wage Tax on the Wage-Rental Ratio: The Long-Run Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production function: ( X_1 = f(K_1, L_1, X_3) )</td>
</tr>
<tr>
<td>( t^* )</td>
</tr>
<tr>
<td>( t_{x1} )</td>
</tr>
<tr>
<td>(a) All ( \sigma )'s = 1.0</td>
</tr>
<tr>
<td>(b) All ( \sigma )'s in the taxed industry = 0.5; other ( \sigma )'s = 1.0</td>
</tr>
<tr>
<td>(c) All ( \sigma )'s in the taxed industry = 0.3; other ( \sigma )'s = 1.0</td>
</tr>
<tr>
<td>(d) ( \sigma^2_{L3} = \sigma^2_{K3} = 0; \sigma^2_{LK} = \sigma^2_{KX} = 1.0 )</td>
</tr>
</tbody>
</table>

NOTE: The \( r^* \) values correspond to the expressions derived in the article (e.g., Equation 14 for \( t_{x1} \)). The tax rate is 1% of the earnings of labor directly employed in the taxed industry.
Both negative and positive \( r^* \)s appear in Table 3, and they get smaller and smaller in absolute value as the elasticity of substitution in the taxed industry drops. Values of \( \sigma^2 \geq 0.5 \) seem to be large enough to make \( r^* \) positive, though, which is surprising because such estimates of \( \sigma \) will be commonly regarded as examples of low substitution elasticities in empirical work. The other specifications replicate this pattern, although the \( r^* \) values differ somewhat. Low substitution elasticities in the taxed sector hurt its primary specific factor in the \( 3 \times 2 \) fgo specification as well (\( r_2^* < 0 \) in two of the three cases, \( \sigma^2 \leq 0.5 \)).

It is also worth noting that in both Tables 2 and 3, for \( t_{L1} \) and \( t_{L2} \), when \( \sigma_{L3} \) in the p.s.i. case is equal to \( \sigma_{L4} \) in the mfo specification, the computed \( r^* \) are different except when all substitution elasticities are unity. This is a reflection of the point made at the end of Section 4.1, that various \( \sigma \)s often are premultiplied by different factor-share terms in the \( r^* \) solutions.

5.3. MODEL EQUIVALENCE AND ELASTICITY RESTRICTIONS

These tables, although providing empirical estimates of tax elasticities, also offer some examples of equivalent models. As the discussion leading to Lemma 1 suggests, the mfo production function, with \( K \) and \( L \) as arguments, can be regarded as “equivalent” to the p.s.i. formulation in Section 2 under certain restrictions on the various elasticities of substitution. Because these are satisfied by the computations in Tables 2 and 3, it is gratifying to observe that for the three taxes being considered, the computed \( r^* \)s have the expected signs in all cases.

For an alternate comparison, the data are rearranged such that the land-labor ratios in the pretax equilibrium are the same in the two specifications: That is, there is a common starting value for \( |\lambda| \) rather than for \( K \), the assumption underlying Tables 2 and 3. Accordingly, the \( K \) and \( L \) numbers in the \( X_3 \) column are transferred to the \( X_1 \) column in Table 1, replacing its row entry for \( X_3 \). The land-labor ratios for \( X_1 \) and \( X_2 \) in the mfo specification, consequently, are further apart than in the earlier tables (a larger value for \( |\lambda| \)), and its effect is highlighted in case (c)\((\sigma^2 = 0.3, t_{L3})\), where, going from Table 3 to Table 4, \( r^* \) drops from \(-.0002 \) (mfo specification) to \(-.0006 \), so labor benefits a good deal more in this situation. By the same token, the gain to capital own-
ers is smaller in Table 4 row (b), but the increase in the rental-wage ratio is the same (.0016) when all substitution elasticities are unity (row a).

The results of a three-input production function, with and without the elasticity restrictions discussed in Section 4.1, are also reported in Table 4 (lower panel). The theoretical discussion suggests that if \( \sigma_{L,1,3}^1 = \sigma_{L,1,3}^1 \), this specification is equivalent to the p.s.i. formulation, but the \( r^* \) estimates, although agreeing in sign, are not identical in the two specifications. Moreover, the computed \( r^* \)s sometimes are rather sensitive to the size of these \( \sigma \)s as, for instance, in Table 4 rows (d) and (e) for \( t_{L,1}^* \). Considering all the computations together, the overall conclusion must be that different versions of the production function for \( X_1 \) do not always produce the same numerical results with this data set.

5.4. SHORT-RUN/LONG-RUN CONSIDERATIONS

The long-run production function (see Section 4.2) is put to work for the computations in Table 5, assuming that one half of \( K_3 \) and \( L_3 \) are directly employed in \( X_1 \). The rental-wage ratio appears to be positively correlated with the elasticity of substitution in the taxed industry, and \( r^* \) is positive for \( t_{L,1} \) as Equation 15 indicates. Row (c) provides an interesting outcome for \( t_{L,2} \): The taxed industry, \( X_2 \), is relatively land intensive, and although there is no formal treatment of this case in Section 4, limited possibilities of input substitution in \( X_2 \) are good for labor because other things equal, there will be a smaller excess supply of labor, and here is an extreme result in this vein, an actual decline in the rental-wage ratio.

To sum up the wide array of computations reported in Tables 2 through 5, the computed tax elasticities do illustrate many of the analytical propositions derived in the article. In some instances, they help in quantifying the qualitative aspects of some results (e.g. Result 3), and they highlight the complications that arise in setting up the empirical counterparts to theoretically equivalent specifications. A common theme running through many of the computations is that analytically equivalent models do not always lead to the same numerical estimates of tax elasticities.
6. CONCLUSIONS

This article has developed a framework for analyzing the role of sector-specific inputs that have a definite value-added component as well as production linkages. They belong to a large and expanding class of inputs ranging from simple, primary commodities to sophisticated high-tech products.

Starting with the classic $3 \times 2$ set-up (McLure 1971), the value-adding process and the contribution of immobile inputs to production throughout the economy are explicitly featured in a small, neoclassical general equilibrium model. The analytical results about tax incidence turn out to be rather different from those in the McLure model, and they seem to be quite sensitive to assumptions about the nature, location, and cross-sector connections of the specific inputs.

The analytical propositions are illustrated, and in some cases refined, with the help of cge examples based on a stylized U.S. data set. Mobility of produced inputs does seem to offset the effects of factor immobility, and several results reflect many aspects of tax models with only mobile factors of production, but tax elasticities computed from the data for the two types of models are rarely the same. By the same token, the Marshallian short-run/long-run distinction, although blurred somewhat by the production linkages modeled here, does not disappear.

In our view, the new results provide ample justification for revisiting a tax model from the 1970s, although some may find this model dated and the issues archaic. Aficionados of the cge approach in particular would ask, Why not set up a realistic model and solve it numerically? Sure, if all one wants is an answer to a particular question, a straight numerical exercise based on cge or other techniques will do the job. Analytical propositions, such as Results 3 and 6, however, are still of interest in taxation theory, and cge models simply cannot generate them. Numerical techniques, including cge, have their place, as in Section 5, for instance. Small, tractable, models of the type deployed here have led to valuable insights into how different taxes operate, and in that capacity, they continue to play a useful role. Besides, there is nothing archaic or outmoded about the examples formalized in this analysis, or the issues of goods mobility and factor immobility, short-run versus long-run, and model equivalence generally, espe-
cially the intertwining of theoretical and empirical aspects highlighted in this article.

NOTES

1. Throughout this article, the terms sector-specific inputs, specific factors, and immobile factors will be used interchangeably. By the same token, why some factors are immobile, or specific to individual sectors, will play no part in the analysis.

2. The elasticity of substitution in $X_i$ is defined as $\sigma_i = \left( a_{K_i} - a_{L_i} \right) / (w^* - r^*)$. The expressions for $R^*$ and $a_{ij}^*$ follow from these definitions and that in the type of production functions specified here, at minimum-unit cost, the share-weighted average of $a_{ij}^*$ in each industry is zero. See Jones (1971b) for more details.

3. Because this stability condition must hold even when $\sigma_2$ and $\sigma_3$ are zero, it may be expected that $(M_1 - S_1) < 0$, and analogously for $(M_2 - S_2)$. This will be useful later on while interpreting some of the results about $t_{11}$ and $t_{12}$.

4. Jones (1971a) pointed out that sufficiently severe factor-market distortions can lead to different industry ranks in physical and value terms so that $[\lambda(p_{x_2} - \theta_{x_2})]$, the first term in the denominator of $r^*$, becomes negative. Because there are no taxes in the initial equilibrium, and we are considering only "small" tax changes, we shall continue to focus on situations in which $D_i > 0$.

5. The partial elasticity of substitution between, say, labor and land in $X_i$ is defined as $\rho_{LK} = (\partial a_{K_L}/\partial w)(a_{K_L}/w)$. Complementarity implies that $\partial a_{K_L}/\partial w < 0$. As Allen (1967) showed, in a three-input production function of the type specified here, at most, only one of the $\sigma$s can be negative.

6. It is assumed first that, in our notation, Solow’s corporate sector corresponds to $X_1$ and the noncorporate sector to $X_2$ and $X_3$ combined. The $X_3$ column is then built up in two steps: First, because the p.s.i. specification does not have any $K$ in $X_3$, corporate capital in Solow’s Table 1 is transferred to $X_1$ (where it will be indirectly used by $X_1$, through the intermediate good); second, roughly 36% of the labor employed in the noncorporate sector is allocated to $X_3$, which is a first approximation of the ratio of $X_1$’s intermediate usage to the value added outside this sector. In terms of the $r^*$ expressions in the article, $[\lambda]$ is overwhelmingly positive, which clinches the sign of $r^*$ for most taxes levied in $X_i$. Given that 75% of the labor is employed in $X_1$ according to these data, a slightly different allocation of labor between $X_1$ and $X_3$ will not alter the results much.

7. This seems appropriate, given the focus of this analysis on specific inputs, although other formulations, such as having the same number of workers or the same land-labor ratios, may be equally valid. See Table 4 for some computations based on equal L-K ratios.

8. This reallocation is consistent with the assumption just discussed about land-labor ratios in Table 4, although it is not based on any structural features or other details of the economy depicted by these data.

REFERENCES


Kul B. Bhatia is professor of economics at the University of Western Ontario, London, Canada, and an associate editor of *Applied Economics*. His most recent publications have appeared in *The Journal of Public Economics* and *Public Finance/Finances Publiques*. His current research interests are integrated general equilibrium analyses of taxes and public expenditures.
INDEX

to

PUBLIC FINANCE REVIEW

Volume 29

Number 1 (January 2001), pp. 1-96
Number 2 (March 2001), pp. 97-180
Number 3 (May 2001), pp. 181-256
Number 4 (July 2001), pp. 257-344
Number 5 (September 2001), pp. 345-416
Number 6 (November 2001), pp. 417-489

Authors:


ANDERS, GARY, see Siegel, D.


Dr. MELLO, LUÍZ R., “Fiscal Decentralization and Borrowing Costs: The Case of Local Government,” 108.


GROSSKOPF, SHAWNA, see Färe, R.


LEE, FITZROY A., “Imperfect Competition and Indirect Tax Structure in a Deregulated Telecommunications Sector,” 419.

LÓPEZ-LABORDA, JULIO, see Badenes, N.

PUBLIC FINANCE REVIEW, Vol. 29 No. 6, November 2001 487-489
© 2001 Sage Publications
Onrubia, Jorge, see Badenes, N.
Ott, Attiat F., see Shadbegian, R. J.
Ruíz-Huerta, Jesús, see Badenes, N.
Salas, Vicente, see Marcuello, C.
Schmidt, Stephen J., see McCarty, T. A.
Shiers, Alden F., see Marlow, M. L.
Sobel, Russell S., see Holcombe, R. G.
Turnbull, Geoffrey K., see Mitias, P. M.
Veall, Michael R., see MacNaughton, A.
Wade, Martcia, see Adams, E. K.
Weber, William L., see Färe, R.

Articles:

“Imperfect Competition and Indirect Tax Structure in a Deregulated Telecommunications Sector,” Lee, 419.

“Implicit Finance in Gambling Expenditures: Australian Evidence on Socioeconomic and Demographic Tax Incidence,” Worthington, 326.


“A Note on Earmarked Taxes,” Hsiung, 223.


“Public Policy Toward Pecuniary Externalities,” Holcombe and Sobel, 304.

“Shadow Prices of Missouri Public Conservation Land,” Färe et al., 444.

“Simplification and Decentralization of the Income Tax,” Badenes et al., 49.


“Tipping and the McQuatters Formula,” MacNaughton and Veall, 99.