Make Sense of Problems and Persevere in Solving Them

ALIGNING CHAPTER 1 TO THE COMMON CORE STATE STANDARDS

The first of the Common Core State Standards for Mathematics (CCSSM) Standards for Mathematical Practice asks that we promote lessons and activities where students must “Make sense of problems and persevere in solving them.” In vibrant mathematics classrooms, students are asked open-ended questions and provided with time and space to explore. Sometimes, students need time to digest expectations of a question and make multiple attempts of problems. This is not always easy to a teacher, who naturally wants to help and guide and to provide hints and clues to her students. This chapter provides strategies for allowing students to be in “the driver’s seat,” taking ownership of the material and being comfortable trying more than one method.
Strategy 1: Help students develop self-control to enhance their thinking and independence as well as to ease your own workload.

What the Research Says

Just a few small changes in your methodology could provide an increase in students’ self-control. The increase of students’ self-control does not release the teacher from his or her outside control. Outside control by the teacher is the basic prerequisite for step-by-step cultivation of self-control. Gradually transfer your control and guidance to students as they develop their own control and feelings of responsibility. In a study about students’ self-control, students reported that their lack of self-control makes them feel uncertain about whether they really reached an educational objective. That’s why students want external control by the teacher, and that’s precisely why teachers need to guide students in realizing self-control.

The research has shown the following:

1. High-achieving students have better self-control than students who have learning weaknesses. However, good students sometimes think that they do not need to practice self-control.

2. Preplanned self-control is hard to observe in students. When self-control is observed, it tends to be more reactive than proactive.

3. The more students are proactive in their self-control, the better they are in reacting with self-control.

4. Girls show a stronger tendency toward self-control than do boys. Boys tend to skip steps of self-control or do it superficially.

5. Within situations of high demands (such as tests), students realize a greater degree of self-control. Self-control during homework tends to be considered superfluous.

6. When teachers control students’ behavior, students tend to adapt to it and refrain from self-control.

7. Teachers’ efforts to encourage students’ self-control focus only on reactive or result-related self-control.
Chapter 1: Make Sense of Problems and Persevere in Solving Them

8. Students know only about techniques of result-related or reactive self-control. These include self-checking, using reference books, using calculators, or verifying the results of a calculation. Low-achieving students tended to mention verifying a calculation as the technique of self-control. Only a few high-achieving students identified making rough estimates as a technique of proactive self-control.

Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics states that “a major goal of school mathematics programs is to create autonomous learners.” Giving students the responsibility for their own self-control is an important step toward allowing students to grow into independent learners. “Students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress.” “Effective learners recognize the importance of reflecting on their thinking and learning from their mistakes.” All of the following applications support this theme and advance the goal of creating autonomous learners.

Aligning to the Common Core State Standards

Teachers are faced with the challenge of navigating the delicate balance between allowing students to struggle (in order to persist) and providing guidance and feedback so they do not give up. Now that “Make sense of problems and persevere in solving them” is an explicit Common Core State Standard, the strategies in this chapter can help students achieve mastery of this standard. As students become more self-aware of how they are thinking and able to analyze problem to persevere, they will increase their confidence and hone their ability to work both alone and with peers. Students who are consistently dependent on the instructor for guidance and review find new problems very difficult and also have difficulty performing well on standardized examinations.

Classroom Applications

Practice continuous methods of self-control such as the following:

- Making rough estimates (Do not trust blindly in the calculator!)
- Using mathematical theorems
- Procedures for drawing representations
- Procedures for graphic representations
- Using templates and measuring instruments

Practical ways for step-by-step improvements in students’ self-control consist of the following:

- Students make mutual comparisons of their answers and solutions strategies.
- When one student presents his or her way of solving a problem, another student should give feedback to the problem-solver.
- Combine assignments with elements of playful self-control. This is suitable particularly for students with learning weaknesses or students with impulsive work habits.

At the end of this strategy, you will find an example of an exercise with such a combination between assignment and playful self-control.

- Give assignments that force students to engage in proactive self-control:
  - Design a task that contains superfluous information.
  - Assign a problem or task that is not solvable or that is solvable only under certain conditions.

With each of these techniques of self-control, a teacher is likely to complain about loss of time. However, the students’ developing self-control abilities will save time in the long run.

**Precautions and Possible Pitfalls**

Strict demands on students to use techniques of self-control incessantly or indiscriminately can backfire. Students who already understand an algorithmic procedure will view the demanded checking as only a mechanical (and therefore meaningless) activity. As a result of this, these students may devalue the self-control.

**Source**

Strategy 2: Encourage students to be mentally active while reading their textbooks.

What the Research Says

When comparing students who are good at understanding what they read with students who are poor at understanding what they read, research shows that good comprehenders are more mentally active than poor comprehenders. Mental activities that characterize good comprehenders include skimming, self-questioning, rereading, inferring, and visualizing. In addition to using such strategies for actively processing the text, good readers tend to coordinate their reading strategies to achieve comprehension.

Teaching to the National Council of Teachers of Mathematics Standards

Teachers of mathematics rely on their textbooks in their day-to-day teaching. Decisions on what to teach and how to teach it are informed by the choice of textbook that students are using. Educators must teach students how to use the textbook effectively. One of the primary functions of the textbook is to provide exercises for students to solve as well as a variety of examples that highlight methods of solution. The approach that a particular author takes to the teaching and learning of mathematics influences the scope and sequence of the topics. In addition, the manner in which various process strands are incorporated into the content strands further magnifies the importance of choosing an appropriate text and using it correctly. Because the National Council of Teachers of Mathematics values both process and content, it is essential that teachers instruct students to use the textbook to compare and contrast solutions to a wide variety of problems. Students should not view the textbook as a source of problems but rather as a source for seeking solutions. Thus, actively engaging a student with a good textbook provides the student with a safety net and affords the student the opportunity to take academic risks in problem solving. Where appropriate, sample exercises should be used to generate discourse on alternative solutions that are available.

Aligning to the Common Core State Standards

Textbook authors and editors are mathematics teachers and mathematicians who take great care in selecting material and
writing explanations. Students who are becoming problem-solvers need resources, and the textbook should be the very first source since it was carefully chosen and presents mathematics in a precise and age-appropriate manner. Since teachers use the textbook as a resource, modeling textbook use is one example of a strategy that will allow students to meet this expectation that all students will “make sense” of mathematical problems. There is likely a similar example in the text or guidance with a concept.

Classroom Applications

It is unfortunate that most students do not use their mathematics textbook in the way authors would like them to be used. Typically, students only use textbooks to complete homework assignments or to prepare for a test. This use shortchanges many students, for the textbook could very well (and often does) provide alternative explanations to a concept explained by the teacher in class. Students would be well advised (and should even be urged) to read the explanatory material covered in class, for it is quite conceivable that a student’s notes are not always complete or truly reliable. Reading a mathematics book is clearly not like reading a novel. The teacher ought to take time out from the normal mathematics instructional program to focus on the way a mathematics textbook ought to be read. By taking a small snippet of time from each of a series of lessons to consider the textbook and how it should be used, the teacher will be making the review (through the textbook) of future topics studied much more effective. The teacher should explain the notation and style of the author and indicate the author’s pedagogical intentions and any other peculiarities that may be appropriate. Teachers should also help students develop the analytical skills for identifying when, why, and how a particular model described in the text fits a particular problem. Students must constantly question their understanding of each idea and look toward the overriding direction or “big picture” of the concepts and units being developed and how they are related to other concepts. Oftentimes, mathematics textbooks offer model solutions to problems. These should also be read in a very active fashion before doing the exercises, even if the student thinks he or she can “fly” through the exercises after or without reading the explanatory material.

Precautions and Possible Pitfalls

The teacher should make a special point of instructing students to read the textbook regularly. In doing so, teachers should
highlight specific aspects of the readings, such as the differences between class instruction and the textbook material (if such exists). The teacher should be aware that there may exist individual reading problems with students in the class that may not manifest themselves in their mathematics achievement. That is, a good mathematics student could be a poor reader. The teacher’s awareness of and sensitivity to these weaknesses are important when considering the task of reading mathematics textbooks.

**Source**

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**Strategy 3: Praise mistakes!**

**What the Research Says**

Mistakes are desirable! Most teachers consider mistakes as something forbidden. They immediately correct mistakes in the text, on the blackboard, on posters, in exercise books, and in every student’s answer. Prohibiting mistakes produces anxiety about making mistakes, and accordingly, students are inhibited. This attitude toward mistakes easily can become a heavy burden, especially when there is time pressure (see Figure 1.1).

<table>
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<th>Teachers’ Answers</th>
<th>Number of Teachers</th>
<th>5</th>
<th>7</th>
<th>12</th>
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<td>$19.00</td>
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<td>Correct</td>
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</table>

In one study, students and teachers (*N* = 38) were asked to solve the following problem within two minutes and to write down the answer.

A customer buys a pocketknife for $6.00. He pays with a $10 bill. Because the owner of the shop does not have enough change, he goes to his neighbor and gets change for the bill. He gives $4.00 in change back to the customer. After the customer left the shop the neighbor came over and said,
“The bill is counterfeit! I want my $10 back!” The shop owner quickly gave a real $10 bill to his neighbor. How much loss does the shop owner sustain if he bought the pocketknife at a price of $5.00 and if he does not count his loss of income?

The distribution of teachers’ answers is equivalent to the students! Teachers gave the following reasons for making their mistakes: time pressure, feeling controlled, being in public, and lack of concentration.

One study (Bangert-Drowns, Kulik, Kulik, & Morgan, 1991) has shown that feedback is most effective when it is provided in a supportive manner, with an emphasis on guiding students to modifying their answers.

Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Communication Standard states that students should “communicate their mathematical thinking coherently and clearly to peers, teachers, and others” and “analyze and evaluate the mathematical thinking and strategies of others.” Teachers who encourage participation will have classrooms filled with students eager to contribute. To foster and maintain this healthy learning environment, teachers need to deal effectively with “wrong answers” in a thoughtful and meaningful way. The feedback a teacher provides to a student is a major factor in the student’s learning process. Feedback should focus on how a student arrived at an answer. Teachers should not dwell on wrong answers but instead highlight the “correct” thinking that the student employed in arriving at an answer. Almost all answers have redeeming qualities, and teachers should seek to find something good in every answer. Teachers should not draw additional attention to a student’s incorrect response. Such actions are humiliating and do not foster an atmosphere where students are encouraged to volunteer, attempt alternative strategies, or take academic risks.

Aligning to the Common Core State Standards

Have you ever heard someone say, “Wow, that was a great mistake!” Probably not, even though we all make mistakes. If we take the time to examine errors along the way, we will develop greater precision in our thinking. Students should work together to find
paths of correct thinking as well as to correct each other when the desired result hasn’t been yet found. The CCSSM encourage students to “detect possible errors by strategically using estimation and other mathematical knowledge.” When students become fluent in this mathematical practice, they “also can step back for an overview and shift perspective,” which is another essential driving force in the thinking of the CCSSM.

Classroom Applications

This kind of stress happens to almost every student in almost every lesson. Therefore, a teacher has to be careful when reacting to students’ errors that result from stress. This works if the teacher reacts by giving small doses of help so that the student can partially solve the problem and has some degree of success experience. In addition, you can use mistakes in creative ways, such as blueprints for student self-correction and as wrong-answer choices for a multiple-choice test!

Creating anxiety in students about making mistakes depends on the teacher’s reaction to mistakes. Teachers generally react on different levels. The first level involves informing the student that a mistake has been made. Subsequent levels usually involve giving substantial or formal help. The teacher’s feedback can assume a variety of forms, from insulting to neutral to encouraging.

Examples of insulting the one who made mistakes:

- “Nonsense! Pay better attention!”
- “Sleepyhead! Your answer is rubbish!”
- “If you open your mouth, just rubbish comes out!”

Examples for neutral marking of mistakes, without evaluating the student who made them:

- “It is not right!”
- “That’s wrong!”
- “There is some mistake!”

Examples of encouraging the one who made a mistake:

- “I am afraid that was not quite correct!”
- “Almost right! Try it again!”
- “Good idea, but, unfortunately not the right direction!”
- “Unfortunately wrong! If you continue thinking about it, you certainly will get the right answer!”
Creative Use of Mistakes

Mistakes can be used to uncover wrong ways of thinking. Therefore, you should give problems that have several plausible solutions. Suggestions for solutions can be reasoned and discussed. Figure 1.2 shows this schematically. Such discussions about how to solve the problem can occur in groups. Problems with several plausible solutions are quite rare. It is also possible to work with mistakes if students know the correct solution. Then, they have to analyze their mistakes. Figures 1.2 and 1.3 show this schematically.

Figure 1.2

Why is Answer 1, 2, or 3 correct?
Why is Answer 1, 2, or 3 wrong?

Figure 1.3

Why is Answer 1 wrong?
How is the wrong answer argued? What was the (wrong) way of thinking? What was mixed up?
Precautions and Possible Pitfalls

Do not wait for students to make mistakes to use errors creatively. It is recommended that you give students problems that are likely to cause errors so you can teach students to analyze them and use errors to improve their future performance. If you wait to use one of the many accidental mistakes that occur, students may interpret the detailed discussion that ensues as insulting or embarrassing the student who made the error, and the class might rebel against the wonderfully beneficial activity of error analysis.

Sources


Strategy 4: Make a lesson more stimulating and interesting by varying the types of questions you ask students.

What the Research Says

Research was conducted to investigate what questions teachers asked and why they asked them. Thirty-six high school teachers from five schools, representing all subject areas, participated in the study. They were asked to give examples of the questions they asked, to explain how they used them, and to tell to whom the questions were addressed. These results, along with findings from previous research by Bloom, Englehart, Furst, Hill, and Krathwohl (1956) and Smith and Meux (1970), led to a system of classifying types of questions teachers ask in the classroom (offered later).
Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Reasoning and Proof Standard suggests that students should “select and use various types of reasoning and methods of proof.” Teachers, by varying their questioning techniques, can lead students to think more creatively and make valuable discoveries as the plot of the lesson unfolds through a thoughtful series of questions and responses. Questions should be constructed to stimulate different forms of thinking. Some (simple) questions merely test recalling of data or procedures while other questions aim to stimulate higher-order thinking and problem solving. Teachers’ questions can shape student learning as the types of questions place emphasis on the process strands that are valued in the teaching of mathematics.

Aligning to the Common Core State Standards

When students become accustomed to repetition and regurgitation, their problem-solving skills are not being developed. In addition to a lack of skill development, students can become disengaged and possibly create classroom management issues while other students “turn off” their cognitive processes. The questions you ask of your students will cause them to think in new ways to solve problems. When teachers consistently model thoughtful questioning techniques, students will learn and duplicate this type of thinking, enhancing their ability to take on new and difficult mathematical problems that are valued in the learning of mathematics.

Classroom Applications

There are many types of questions to use as well as many to avoid. Much of the nondrill topics in mathematics require understanding. When a topic that requires thought and deduction is being considered, it is wise to ask lots of questions. Each question should be succinct and structured to lead the students through a mathematical development or argument. One example is the sort of questioning that a teacher might use when guiding a student through a geometry proof. Here the questioning can take the form of factual questions or questions that have no definite answer but that require a judgment to be made. Following is the list alluded to earlier.
Cognitive Questions

1. Recalling data, task procedures, values, or knowledge. This category includes naming, classifying, reading out loud, providing known definitions, and observing.

2. Making simple deductions, usually based on data that have been provided. This category includes comparing, giving simple descriptions and interpretations, and giving examples of principles.

3. Giving reasons, hypotheses, causes, or motives that were not taught in the lesson.


5. Evaluating one’s own work, a topic, or a set of values.

Speculative, Affective, and Management Questions

1. Making speculations, intuitive guesses, offering creative ideas or approaches and open-ended questions (that have more than one right answer and permit a wide range of responses).

2. Encouraging expressions of empathy and feelings.

3. Managing individuals, groups, or the entire class. This category includes checking that students understand a task, seeking compliance, controlling a situation, and directing students’ attention.

Precautions and Possible Pitfalls

Even good questions can lose their value if they are overused. Avoid asking ambiguous questions and questions requiring only one-word answers, such as yes/no questions. To focus on a questioning style as indicated previously without proper concern for the subject matter would be a misuse of this strategy.

Sources


### Strategy 5: Use a variety of strategies to encourage students to ask questions about difficult assignments.

#### What the Research Says

Several approaches have been identified that can help to overcome students’ reluctance to ask questions:

1. Avoid giving students the impression that the reasons for difficulties are their own.
2. In cases where students have difficulties with problems, do not indicate that the problem was simple.
3. Give external reasons for students’ difficulties.

Research has shown that people can handle their neediness better if they can attribute the reasons for their neediness to external causes. One study investigated asking questions as a kind of neediness. Participants were 24 girls and 24 boys, with a mean age of 14 years.

Students were confronted with the following situation. They got an unformatted text that included typing mistakes. Students had to format the text according to a given pattern.

The results showed the following:

1. Students showed the most willingness to ask questions when they could hold external circumstances responsible for their neediness.
2. Students’ willingness to ask questions decreased when they had the impression that the person they asked blamed them for the difficulty. In this case, if the students asked a question, it would hurt their self-esteem.
3. Students avoided asking questions if the person they asked indicated the task was simple.
Teaching to the National Council of Teachers of Mathematics Standards

In the same manner that the National Council of Teachers of Mathematics Communication Standard requires students to think and speak mathematically, it is of equal or greater importance for teachers to create an atmosphere where students are encouraged to take risks as they pose questions and formulate responses to challenging problems in mathematics. Oftentimes individual students feel that they are alone in their inability to solve difficult problems when, in fact, it is a feeling shared by the entire class. Students are thus afraid to voice their uncertainty. A Research Companion to Principles and Standards for School Mathematics cautions teachers against silencing and marginalizing questions and responses from students that may be indicative of difficulty in understanding the concepts of a lesson. Instead, teachers should use the strategies outlined later to respond appropriately to student questions.

In proactively supporting students’ mathematical learning, the teacher necessarily has to treat students’ contributions to the classroom discussion differentially. The decisions the teacher makes might be entirely justifiable from a mathematical point of view. However, in differentiating between students’ contributions, the teacher implicitly communicates to students that certain opinions and ways of reasoning are particularly valued and others are less valued. Every question should be treated as a valued communication between students in the class and the teacher.

Aligning to the Common Core State Standards

Difficult assignments are essential for student learning but can also produce reactions that hinder those same learning opportunities. This chapter provides specific strategies for encouraging students and guiding them through the more challenging examples and problems. The way we speak and listen to students can enhance their problem-solving experience or hinder it. To master the material in CCSSM, we must enhance their experience as much as possible!

Classroom Applications

Teachers should explicitly and implicitly encourage students to ask questions. Asking questions is not easy for students in many cases. Sometimes even simple questions require both a minimum of knowledge/understanding and courage. Teachers should help
students feel that there are no such things as silly questions although teachers sometimes give silly answers! Asking questions is one of the most valuable skills a person can develop. Teachers can say that “silly” questions are often the very best questions. Teachers should do the following:

**Make positive comments about students’ questions.**

Examples

“Good question!”

“Instead of getting grades for good answers, you should get grades for good questions!”

“Your questions show that you’ve thought about this a lot.”

“Very interesting question!”

**Encourage students to ask questions by emphasizing the difficulties of the task or of the working conditions. Or, give an understatement of your own abilities.**

Examples

“Some aspects of this problem are hidden. Consequently, you might have some difficulties.”

“We never even talked about some of the steps needed to solve this problem.”

“I didn’t even see this problem.”

“Even today I have to struggle when asking questions in public.”

**If students begin to attribute difficulties to their own lack of ability, try to direct their attention to the external difficulties.**

Examples

“Make sure you pay careful attention to the difficult parts of this problem.”

“This is a new type of problem. We haven’t discussed it yet.”

“Do not expect your brain to work very quickly. It has been a long day.”
Do not express doubt about students’ capabilities or skills.

Negative Examples

“I already answered that question three times.”

“Listen carefully to what I say!”

Positive Examples

“When students ask me a question a third time that tells me that something has gone wrong with my explanation.”

“Okay! We covered a lot of facts—maybe too many.”

“Sometimes I explain things too quickly.”

Precautions and Possible Pitfalls

Beware of possible backfire! When explaining an assignment’s difficulties by external circumstances (very abstract, complex, obscure, or obtuse, or pressure for time or application of a very rarely used technique, etc.), you might encourage students; however, you can also confirm students’ opinion that the assignment is too difficult anyway. In that case, students would not be encouraged but would feel justified in stopping work on the problem, and questioning will cease!

Sources


Strategy 6: Use a question-asking checklist and an evaluation notebook to help students become better learners.

What the Research Says

Numerous studies have demonstrated that students often do not know what they don’t know. Research has also shown that
students can become aware of their strengths and weaknesses as learners, and they can learn to take greater control over their own academic performance.

One such study was conducted with 64 students from two 9th-grade science classes and one 11th-grade biology class. There were four phases to this study:

- **Phase 1: Exploratory.** This phase lasted four weeks and involved getting to know the students and seeking their consent for cooperation and participation in the rest of the study.
- **Phase 2: Awareness.** This phase lasted five weeks for Grade 9 and three weeks for Grade 11. During this phase, students began thinking about themselves as learners. They reflected on their attitudes, learning difficulties, and strategies for overcoming these difficulties.
- **Phase 3: Participation.** This phase lasted seven weeks for Grade 9 and six weeks for Grade 11. Students began using the Question-Asking Checklist and the Evaluation Notebook. The teachers gave students a considerable amount of help using these materials during this phase.
- **Phase 4: Responsibility-Control.** This phase lasted for seven weeks for Grade 9 and three weeks for Grade 11. During this phase, the teachers’ role virtually ceased and students used the materials on their own. Teachers monitored student behavior and attitudes and intervened only as needed.

**Materials**

- **Question-Asking Checklist.** There were 10 different categories, each of which had its own icon and set of questions. The 10 categories were topic, detail, task, approach, change in knowledge, increase understanding, progress, completion, satisfaction, and future use of knowledge. Questions for the approach category were “How will I approach the task?” “How hard will it be?” “How long will it take?” “Is there another way of doing it?” “Why am I doing the task?” “What will I get from it?” and “What will I make of the result?”
- **Evaluation Notebook.** Students evaluated their use of the questions from the Question-Asking Checklist for most of their science lessons during Phases 3 and 4 and recorded the results in this notebook. Data were collected from 15 different sources, including notebooks, classroom observations, audio and video recordings of lessons, interviews, questionnaires,
and teacher-made tests. The results showed that at the beginning, neither the 9th nor the 11th graders were clear about their learning difficulties, and they did not have strategies for overcoming them. During Phase 3, students became more aware of themselves as learners. During Phase 4, students improved in their ability to control their own learning.

**Teaching to the National Council of Teachers of Mathematics Standards**

The National Council of Teachers of Mathematics Learning Principle states that “learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills.” Although much emphasis has been placed on teacher questioning techniques, there should be equal emphasis on student self-questioning and self-evaluation. The following application provides a framework for student self-evaluation that can help students become less reliant on teacher assessment and provide their own formative assessment. This advances the goal of making students autonomous learners.

**Aligning to the Common Core State Standards**

According to the authors of the CCSSM, mathematically proficient students “monitor and evaluate their progress and change course if necessary.” Sometimes this process is intuitive to students, but the Question-Asking Checklist and Evaluation Notebook described in this chapter make it concrete and real to students who need support in this area. Rather than asking students to reflect in a general way, checklists and guided notebooks provide tangible tools to train students in persevering and solving problems in mathematics.

**Classroom Applications**

Such a checklist can also play an important role in the mathematics classroom when approaching a problem for solution. This list might include such questions as “What does the problem call for?” “What am I to look for?” “What am I given?” “What do I know about the problem situation from prior experience?” “Where have I solved a similar or analogous problem?” “What is the relationship between
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the given data and that which is to be found?” “Is my answer reasonable?” and so on. The actual questions for the proposed problem must be consistent with the students’ knowledge base and ability. By keeping a record and being forced to verbalize thoughts and actions, students come a long way toward getting involved in and reflecting on their own real problem-solving behaviors.

Precautions and Possible Pitfalls

Teachers must be careful not to impose their checklist on the students. They must have students formulate their own list so that the items are meaningful and useful to the students. Students should have a feeling of ownership of the list, and questions should be adapted to the specific problem.

Sources


Strategy 7: Find out why students rate a mathematical task as difficult so you can increase the difficulty of exercises and tests more effectively.

What the Research Says

Systematically increasing the difficulty of exercises and tests helps reinforce students’ motivation to learn and also produces better test results. However, research shows that teachers and researchers perceive difficulty very differently from students. Consequently, it is important for teachers to find out how their students feel about the difficulty of mathematical tasks so they can construct tasks with the appropriate increase in difficulty. A group of 40 ninth-grade students (two classes) took a test consisting of four sets of eight arithmetical tasks of increasing difficulties. The test lasted 35 minutes. Every set formed one page. Three grades of difficulty were used: easy (E), medium (M), and difficult (D). Students were divided into Groups A and B. Their tasks were the same; however, the increasing difficulty of
the tasks changed. Group A had to solve the three main sets in the sequence EMD, while Group B had to solve the three main sets in the sequence MED. At the end of every page, students estimated their success (“How many of the eight tasks do you think you solved correctly?”). In addition, students used a seven-stage scale to rate the difficulty of the set as well as the fun they had on each page and the success they expected for the next page. The results showed that Group A (EMD) had significantly more correct solutions than did Group B (MED). Students in Group B rated the level of difficulty higher than did students in Group A. Whereas teachers and researchers described tasks as medium difficulty when 50% are solved, students rated tasks as medium difficulty when they were 82% solved and rated them as difficult when 71% were solved! Students who thought they had correctly solved all eight tasks in a set rated most of the tasks as easy.

**Teaching to the National Council of Teachers of Mathematics Standards**

The National Council of Teachers of Mathematics Learning Principle states that “students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.”

To properly assess what degree of learning has taken place, students must demonstrate that they have a firm grasp of procedural and conceptual knowledge of a topic. If the Learning Principle calls for the building of knowledge from experience and prior knowledge, the assessment must be constructed so that there is a “ramping up” of questions from easy to medium to hard. An examination that is not crafted in such a manner provides a psychological challenge that may interfere with the main goal of assessing a student’s mathematical proficiency. Teachers can use a variety of methods to obtain this information; however, a careful analysis of student performance on homework questions can provide very useful information about the degree of difficulty of items. This information, augmented by student questionnaires and interviews, will give the teacher valuable insight into levels of difficulty for assessment purposes.

**Aligning to the Common Core State Standards**

Gaining feedback from students is an underused resource in mathematics education. How better to help students persist when
faced with difficulty than to diagnose precisely why they feel a task is “hard”! Asking for student input will increase their dedication to mathematical processes in two ways: First, they will feel that you are listening to their thoughts and concerns, building trust and a positive relationship, and second, you will be able to identify ways to help them through what they consider to be the “hard part.” This leads to a win-win for both you and the student. As an added advantage, you will also have data to show that you are monitoring and revising your instruction based on student needs.

Classroom Applications

If teachers choose to increase the degree of difficulty, they need information about students’ feelings and perceptions. To get this information, teachers can create a test and link it to questions that have students rate the difficulty of each item on a seven-stage scale such as the one in Figure 1.4.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>Medium difficulty</td>
<td>Difficult</td>
<td>Pretty difficult</td>
<td>Very difficult</td>
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</tbody>
</table>

Similarly, teachers can investigate students’ enjoyment and their expectations. Sometimes it is wise not to make assumptions about how students feel or to trust one’s own theories or sixth sense about students’ reactions. Instead, teachers should ask the students themselves. One way to do this is through interviewing students. Another way is to administer a questionnaire and obtain statistical data on students’ feelings about tasks, which provides objective information that can help the teacher design tasks consistent with students’ abilities as well as their motivation. Students might also appreciate having their feelings about academic tasks examined.

Precautions and Possible Pitfalls

Do not ask students about their feelings about task difficulty and enjoyment too often. Otherwise, you are likely to get
a distorted impression of the students’ estimation of an assignment’s difficulty, and students may deceive you to make exercises and/or tests easier!

Sources


Strategy 8: Teach students to ask themselves questions about what they already know about a problem or task they are working on.

What the Research Says

Research was conducted comparing the effectiveness of two types of student self-generated questions: (1) questions designed to enhance understanding of connections of ideas within a lesson and (2) questions designed to access students’ prior knowledge/experience and to promote understanding of the connections between that prior knowledge and material in the lesson. All students were trained to give explanations. Teachers were trained by the researcher to teach students how to ask these types of questions and how to give explanations. As part of their regular classwork, students learned, practiced, and were tested on their self-questioning. The results showed that although both types of self-questions led students to develop more complex knowledge, the prior knowledge questions enhanced content learning more effectively.

Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Problem Solving Standard calls for students to “apply and adapt a variety of appropriate strategies to solve problems.” Students who access prior knowledge can choose among successful strategies employed in the solution of similar problems. This, coupled with
their willingness to approach problem solving through a variety of perspectives, should greatly enhance their likelihood of success. When delivering classroom instruction, teachers should stress “asking the right question” so that students can model this on their own. Employing the Socratic method, or “Teaching by Questioning,” embeds in students’ minds the importance of taking a step back to analyze a problem and asking themselves some important questions that are key to formulating a solution.

### Aligning to the Common Core State Standards

The first of the Standards for Mathematical Practice is titled “Make sense of problems and persevere in solving them.” Application of this strategy will allow students to use what they already know to make sense of a new problem. The elaboration on this practice explains that “mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’ They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.” By prioritizing this strategy, your students will be better prepared to address the fewer, higher, clearer expectations set forth by the CCSSM content standards.

### Classroom Applications

Teach students to question themselves about what they already know about a topic and how this knowledge relates to the current problem/situation. For example, “What do I know about this type of problem? How have I solved problems like this before?” Then teach them to ask themselves how this information applies to the current problem or situation. For example, “How can I use that approach in this situation?”

Student-generated self-questions are very effective thinking and learning tools. These self-questions need to be formulated for the specific problem or task the student is working on. Student-generated questions are superior to teacher-generated questions that are given to students to use. Not all self-questions are equally valuable. When doing problem solving in a mathematics course, there are many times students should ask themselves questions. This form of silent questioning is an excellent way of guiding oneself through the solution of a problem. This sort of self-questioning replaces the typical teacher
questioning and begins to develop a problem-solving independence in the student. Quite normal people can—and ought to—talk to themselves in an effort to find the most efficient method to solve a mathematics problem.

**Precautions and Possible Pitfalls**

Questions generated by the students themselves are more effective than are questions provided to them by the teacher, and although student questions are often unpolished and may even sound inaccurate, they understand them and often resent having them sharpened by the teacher.

**Source**


**Strategy 9: Structure teaching of mathematical concepts and skills around problems to be solved, using a problem-centered or problem-based approach to learning.**

**What the Research Says**

Problem-centered or problem-based learning is becoming recognized as an outstanding way of teaching both content and problem-solving skills. One study compared six classes that received problem-centered mathematics instruction for two years with students who received problem-centered mathematics instruction for one year and with students who received traditional textbook-based instruction. Researchers examined students’ performance on standardized achievement tests and investigated students’ personal goals and beliefs about the reasons for their success in mathematics. The results showed that students who received problem-centered instruction for two years demonstrated significantly higher mathematics achievement than did traditionally instructed students, in terms of both their proficiency in solving problems and their conceptual understanding. In addition,
problem-centered-learning students had stronger beliefs than did traditional students about the importance of finding not only different ways of solving problems but also the importance of finding their own ways of solving problems. Students who received problem-centered mathematics instruction for only one year and then returned to textbook teaching performed at levels comparable with the textbook-instruction-only students. Consequently, to achieve meaningful benefits from the problem-centered approach, students should receive more than one year of instruction using this form of teaching. This problem-centered approach has become standard instructional practice in many medical school programs.

Teaching to the National Council of Teachers of Mathematics Standards

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics states that

Students should view the difficulty of complex mathematical investigations as a worthwhile challenge rather than an excuse to give up. Even when a mathematical task is difficult, it can be engaging and rewarding. When students work hard to solve a difficult problem or to understand a complex idea, they experience a very special feeling of accomplishment, which in turn leads to a willingness to continue and extend their engagement with mathematics. As the following application suggests, introducing a topic with a provocative problem creates an interest that is motivational and gives purpose to the day’s lesson.

Aligning to the Common Core State Standards

The essential message in this chapter is “practice, practice, practice!” As noted in the research, students who are consistently taught using a problem-centered approach achieve at significantly higher levels than do peers who were provided with “direct instruction,” “chalk and talk,” or “textbook-based” instruction. This technique will take time to refine; however, you will find that with practice on your part and the part of your students, the benefits will astound you. Give your students every opportunity to construct new knowledge by
providing them with rich problems. The key to encouraging critical thinkers and problem-solvers is to create a classroom where students are engaged daily in rigorous mathematical problem situations. The CCSSM find that “as they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.”

**Classroom Applications**

Instead of starting a unit by using the textbook and telling students about a mathematical topic and explaining and demonstrating various concepts, problems, and solution methods, start by giving students a meaningful problem to solve. Problem-centered or problem-based learning is a method of teaching that involves using ill-structured, real-world problems as the context for learning basic content through in-depth investigations. To solve the problem, students will need to learn specific mathematical concepts and solution strategies. Teachers give students only enough information to enable them to begin their inquiry. They never give students enough information to actually solve a problem. With this instructional method, students cannot simply solve problems by applying a particular formula. There must be student reasoning and inquiry. Often there is more than one way to solve a problem. Students will learn important concepts and skills through mathematical inquiry in a meaningful context. The teacher’s role is to be a coach, mentor, or tutor who guides students in their inquiry and helps them develop and understand their own thinking.

One topic that lends itself to this sort of problem-solving investigation is the consideration of maxima and minima. For example, students could seek to maximize an area with a given perimeter. They could consider the shape as a variable. They may have it as a rectangle or a polygon. They may even consider a circle with this given perimeter to see how it compares with the other shapes. This topic can be considered before the study of calculus by inspecting the turning point of a parabola or by merely inspecting extremes to see the behavior of the variable.

**Precautions and Possible Pitfalls**

Problems must be at an appropriate level of complexity. In addition, students must have appropriate prior knowledge so they
Know or can figure out what they need to learn to solve the problem. The topic selected must be appropriate for the ability level of the students, and, above all, it must garner the proper interest among the students. Without these considerations, the problems will not serve the desired goals, mentioned earlier.

**Sources**


**NOTES**


