## Measurement Scales

Terms: nominal, ordinal, interval, ratio, continuous, discrete, real limits
Symbols: LL, UL

## Learning Objectives

- Define a scale of measurement
- Classify data according to their scale of measurement
- Distinguish between discrete and continuous variables
- Establish real limits for continuously scored data


## What Is Measurement?

Measurement is the process of assigning numbers to the observations on some variable. However, not all numbers have the same numeric properties. For example, one football player may wear a jersey with No. 7 on it, and a second player may wear a jersey with No. 18 on it. However, you would not say that the second player has 11 points more of any trait than the first player. The jersey numbers simply don't mean that. Or you may earn the top score on a statistics test, your friend Lauren the second highest score, and your friend Sidney the third highest score. However, you cannot assume that the score difference between you and Lauren is the same as the score difference between Lauren and Sidney. Again, the rankings simply don't mean that. Or one person may score 75 on an intelligence test, and a second person may score 150 on the same intelligence test. While we would agree that the second person is probably considerably more intelligent than the first one, we cannot say that the second person is twice as intelligent as the first person. Again, the scores simply don't mean that.

## Scales of Measurement

What, then, can we say about each of these situations? What we can say depends on the scale on which the data were measured. Stevens (1946) suggested that variables are measured on one of four scales: nominal, ordinal, interval, or ratio. Each of these scales allows us to draw different conclusions about the meaning of subjects' scores. Furthermore, the statistics that can be calculated on the data are partially dependent on the measurement scale in the data. Thus, before we can select and compute statistics, we need to know the scale on which the data were measured.

## Nominal Scale

A nominal scale classifies cases into categories. For that reason, it is also sometimes called a categorical scale. Here are some examples:

$$
\begin{aligned}
& \mathrm{m}=\text { male, } \mathrm{f}=\text { female } \\
& \mathrm{l}=\text { married, } 2 \text { = divorced, } 3=\text { separated, } 4=\text { never married } \\
& \text { tel }=\text { owns a telephone, notel }=\text { does not own a telephone }
\end{aligned}
$$

The nominal scale is the lowest level of measurement. It does not measure how much of a measured trait a person possesses. It merely categorizes the person as a "this" or a "that." Even when numbers are used to represent the categories, they are designations only, much like social security numbers or the numbers on a football jersey. Because the designations have no numeric meaning, it makes no sense to perform arithmetic operations on them. For example, you would learn nothing by knowing the average telephone number for the members of your statistics class or the average jersey number for the members of a football team.

Most of the statistics that you will use throughout this textbook require that the numbers have numeric meaning. Thus, most of the procedures that you will learn will not be appropriate for data that are measured on only a nominal level. However, certain statistical procedures have been developed to analyze data in a nominal form. For example, the chi-square statistic,

There are two groups of people in the world: Those who believe that the world can be divided into two groups of people and those who don't.

## Ordinal Scale

An ordinal scale ranks people according to the degree to which they possess some measured trait. Persons are first measured on some attribute (e.g., height). Then, they are assigned ranks according to how much of the attribute they possess. Here are some examples:

$$
1=\text { tallest, } 2=\text { second tallest }, 3=\text { third tallest, and so on. }
$$

$1=$ highest GPA (grade point average), $2=$ second highest GPA, $3=$ third highest GPA, and so on.
$1=$ fastest runner, $2=$ second fastest runner, $3=$ third fastest runner, and so on.
To create ranks, place the scores in order by value (usually descending) and then assign the highest score a rank of 1 , the second highest score a rank of 2 , and so on. Table 2.1 presents scores and ranks for 10 students on a 100 -item statistics exam given as a pretest before the course began.

With an ordinal scale, the difference in test scores between two adjacent ranks may not be the same as the difference in test scores between any other two adjacent ranks. Compare the test scores between adjacent ranks in Table 2.1. The test score difference between the first two ranks is 1 point

| Table 2.1 Statistics Test Scores and Ranks |  |
| :---: | :---: |
| Score | Rank |
| 73 | 1 |
| 72 | 2 |
| 60 | 3 |
| 59 | 4 |
| 57 | 5 |
| 56 | 6 |
| 52 | 7 |
| 48 | 8 |
| 36 | 9 |
| 28 | 10 |

( $73-72$ ), but between the next two ranks, the difference in test scores is 12 points ( $72-60$ ). Clearly, certain aspects of relative performance are lost when scores are expressed as ranks.

Most of the statistics used throughout this textbook require a scale in which adjacent intervals on the measurement scale imply equal intervals between adjacent scores throughout the scale. Because this is not the case for ordinally scaled data, most of the statistics will not be appropriate for data that are measured on an ordinal scale. However, ranks do tell us more than nominal classifications. A higher-ranked person does have more of the measured trait, even if distance on the underlying scores is inconsistent. Therefore, certain statistical procedures have been developed to analyze data when they are in ordinal form. For example, a Spearman rho coefficient measures the degree of relationship between two sets of ranks.

## Interval Scale

With an interval scale, the distances between adjacent scores are equal and consistent throughout the scale. Equal intervals on the scale imply equal amounts of the variable being measured. For this reason, the interval scale is sometimes referred to as the equal-interval scale. Here are some examples:

Scores on the final exam in this course
Scores on an intelligence test
Degrees Fahrenheit ( ${ }^{( } \mathrm{F}$ ) or Celsius $\left({ }^{\circ} \mathrm{C}\right)$


Scores on certain personality or career interest tests
Because interval scales are consistent throughout the scale, it makes sense to compare scores by adding or subtracting them. For example, an intelligence score of 120 is 10 points higher than an intelligence score of 110 , just as an intelligence score of 60 is 10 points higher than an intelligence score of 50. In either case, the higher-scoring person has 10 more points of scaled intelligence than the lower-scoring person.

However, interval scales have no absolute zero point-the point at which a person would have none of the measured attribute. That is, even if a person scored 0 on an intelligence test, we would not say that the person has no intelligence. This is because the test's starting point is fixed and arbitrary, whereas the starting point of the actual trait itself-in this case, intelligence-is unknown. This is typically the case in psychological or educational measurement. Similarly, $0^{\circ} \mathrm{F}$ or $0^{\circ} \mathrm{C}$ does not indicate a lack of temperature-it refers to a low temperature.

Because of the lack of an absolute zero point in an interval measurement scale, it makes no sense to compare interval scores by multiplying or dividing them. Although a person who scores 120 on an intelligence test has scored twice as high as the person who scores 60 , the person who scores 120 does not have twice as much intelligence as the person who scores 60 .

Most of the statistics that you will learn throughout this textbook require at least an interval measurement scale. Fortunately, most data in social science research are also interval scaled. Therefore, the statistics that you will learn are ideally suited for most educational and psychological data.

One controversy in educational and psychological research is whether personality and career interest tests are interval scaled or only ordinal scaled. This is because individual test items typically ask test takers to use a noninterval scale. For example, test takers might be asked to rate the frequency with which they feel that "life has no meaning" with the following scale: never, occasionally, often, or always. Or the scale might ask test takers to rate the degree to which they agree with the statement "I enjoy working out-of-doors" with the following scale: strongly agree, slightly agree, neither agree nor disagree, slightly disagree, or strongly disagree. We cannot say that the difference in frequency between never and occasionally is the same as the difference in frequency between often and always. Similarly, we cannot say that the difference in degree between strongly agree and slightly agree is the same as the difference in degree between slightly disagree and strongly disagree. Therefore, the scale appears to be ordinal.

On the other hand, at the completion of the personality or career interest test, the test taker receives a score, much like the score you might receive on a test in this statistics course. Moreover, the scores are typically normed against a large number of test takers. This gives any one person's score both position and distance when compared with the known distribution of all scores. These features are typical of an interval scale, not an ordinal scale. Because the scores contain features of both ordinal and interval scales, the debate over the appropriate level of measurement, and hence the appropriate statistics to use to describe the scores, is ongoing.

## Ratio Scale

A ratio scale is like an interval scale, in that the distance between adjacent scores is equal throughout the distribution. However, unlike an interval scale, in a ratio scale there is an absolute zero point. That is, there is a point at which a person does not have any of the measured traits. Because the trait's starting point is known, the scale reflects that zero point.

Ratio measurement typically applies to measures in the physical sciences. Here are some examples:
Height
Weight
Distance
Time
Temperature measured in degrees Kelvin
Because there is an absolute zero point, it makes sense to compare scores by multiplying or dividing them. For example, a person who weighs 110 lb not only weighs 55 lb more than a person who weighs 55 lb but also is twice as heavy. A person who makes a standing broad jump of 3 ft not only jumps 3 ft less than one who jumps 6 ft but also jumps only half as far. On the temperature scale of Kelvin, 0 degrees means an absolute lack of energy-no heat at all!

Although most data in psychology and education are interval scaled, some are ratio scaled. For example, measures of reaction time or physical performance are ratio scaled. Any statistic that is appropriate for interval-scaled data is also appropriate for ratio-scaled data.

Finally, it is possible to measure data on more than one scale. That is, higher-level data can be measured on a lower-level scale. For example, your interval score on a statistics test can be nominally categorized as either pass or fail. However, lower-level data cannot be measured on a higher-level scale. For example, your sex (male or female) cannot be ranked. It can only be nominally categorized.

## Check Yourself!

Give examples of nominal, ordinal, interval, and ratio data. Then, convert your ratio data example into interval, ordinal, and nominal scales.

## Practice

1. Indicate the first letter ( $\mathrm{N}, \mathrm{O}, \mathrm{I}, \mathrm{R}$ ) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:

| Measure | Highest Scale |
| :--- | :--- |
| a. Feet of snow | - |
| b. Brands of carbonated soft drinks | - |
| c. Class rank at graduation | - |
| d. GPA |  |
| e. Speed of a baseball pitch |  |

## (Continued)

2. Indicate the first letter ( $\mathrm{N}, \mathrm{O}, \mathrm{I}, \mathrm{R}$ ) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:

| Measure |
| :--- |
| a. Eye color |
| b. Time taken to solve a puzzle |
| c. Genre of favorite television program |
| d. Level of depression |
| e. Position in a starting lineup |
| f. Number of angels that can fit on the head of a pin |
| (Oops ... angels may not be subject to earthly |
| measurement constraints, so you may skip this one.) |

3. Indicate the first letter ( $\mathrm{N}, \mathrm{O}, \mathrm{I}, \mathrm{R}$ ) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:

4. Indicate the first letter (N, O, I, R) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:

| Measure | Highest Scale |
| :--- | :--- |
| a. Favorite radio station |  |
| b. How much sleep you had last night |  |
| c. How many calories you ate today |  |
| d. Athletic ability |  |
| e. Position in a graduation processional |  |

5. Indicate the first letter ( $\mathrm{N}, \mathrm{O}, \mathrm{I}, \mathrm{R}$ ) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:
Measure $\quad$ Highest Scale
a. Literature genres
b. Relative academic position in one's graduating class

| Measure | Highest Scale |
| :--- | :---: |
| c. Hair length | - |
| d. Job salary | - |
| e. Self-confidence | - |

6. Indicate the first letter ( $\mathrm{N}, \mathrm{O}, \mathrm{I}, \mathrm{R}$ ) of the highest possible scale for each of the following measures, where N is the lowest and R is the highest:

| Measure | Highest Scale |
| :--- | :---: |
| a. Type of disease | - |
| b. Business profit | - |
| c. Pass or fail status |  |
| d. Conscientiousness |  |
| e. Car brands |  |

## Continuous Versus Discrete Variables

Continuous variables are variables whose values theoretically could fall anywhere between adjacent scale units. The data that are measured on a ratio scale are always continuously scored. A person's height or weight or even the time a person spends talking on the phone can fall anywhere between the scale units.

Some interval scale data are continuous variables. For example, although people do not score fractional points on IQ tests, depression inventories, or measures of talkativeness, they theoretically could score fractional points if the tests were so graded.

Discrete data, on the other hand, are values that cannot even theoretically fall between adjacent scale units. Some interval scale data are discrete. Examples are the number of blue ribbons won, number of children in a family, or number of photographs taken. With discrete interval data, we can perform all the arithmetic operations on the data that we would with any other interval-scaled set of scores. That is, we can speak of twice as many blue ribbons won, the average number of children in families, or half as many photographs taken. At the same time, we recognize that individual scores cannot fall between the scale units. That is, no single person can earn a partial ribbon, have a partial child, or take a partial photograph.

Bumper sticker: Statisticians do it both continuously and discretely.

## Check Yourself!

Is the number of courses students register for in a semester a discrete variable or a continuous variable? Is the GPA that students earn in those courses a discrete variable or a continuous variable?

## Practice

7. Are the following variables discrete or continuous? Mark "D" or "C" to indicate your answer:

| Measure | Variable Type |
| :--- | :--- |
| a. Inches of rainfall |  |
| b. GPA |  |
| c. Speed of a baseball pitch |  |
| d. Time taken to solve a puzzle |  |
| e. Level of depression |  |
| f. Number of angels that can fit on the head of a pin (Oops ... |  |
| how'd that get in here again? You may skip this one.) |  |

8. Are the following variables discrete or continuous? Mark "D" or "C" to indicate your answer:

| Measure | Variable Type |
| :--- | :--- |
| a. Hair length | - |
| b. Yearly income |  |
| c. How much sleep you had last night |  |
| d. How many calories you ate today |  |
| e. Athletic ability |  |

9. Are the following variables discrete or continuous? Mark "D" or "C" to indicate your answer:

| Measure | Variable Type |
| :--- | :--- |
| a. Number of TVs in the household | - |
| b. Level of extraversion | - |
| c. Color saturation level | - |
| d. How tired you feel right now | - |
| e. How many awards you won as a child |  |

10. Are the following variables discrete or continuous? Mark "D" or "C" to indicate your answer:

| Measure | Variable Type |
| :--- | :--- |
| a. Amount of irritation you felt today |  |
| b. Number of times you felt irritated today |  |
| c. How many different ice cream flavors you have tasted |  |
| d. How much weight you have gained or lost in the past year |  |
| e. How badly you need a vacation right now |  |

## Real Limits

For continuous variables, scores can theoretically fall anywhere between scale units, even if actual scores fall only at specified locations along the scale. In other words, even though a continuous scale uses the units of 1,2 , and 3 , it is theoretically possible to have a score between 1 and 2 . As an example, imagine that a syllabus for a Statistics course says that students who have a mean (which is a statistics way of saying average) score above a 90 on three tests, whose possible scores range from 0 to 100, will receive an A. David earns an 85, 86, and 98 on those three tests, giving him a mean test score of approximately 89.67 (we will cover how to compute a mean in Module 5). His mean falls right between 89 and 90 ! So should David receive an A? You might be inclined to say "No" because 89.67 is less than 90 . However, because this scale is continuous, it is important to consider the score's real limits.

On a continuous scale, each score point (e.g., 90) actually refers to a range of possible scores that include all of the values from 90 to the adjacent scores. In our current example, the adjacent scores are 89 and 91 . Real limits are set by using half the scale's unit. Thus, the real limits of any particular score are the score plus or minus (symbolized as $\pm$ ) one-half scale unit. In our current example, the test scores were scaled by 1 point, so the real limit for each point is $\pm 0.5$. That means the cutoff of 90 actually corresponds to all the values that could fall between 89.5 and 90.5 . The lower real limit ( $\mathbf{L L}$ ) is one-half unit below the score (in this case, 89.5), and the upper real limit $(U L)$ is one-half unit above the score (in this case, 90.5). No scores actually fall at the real limits. Returning to our example, the score of 90 on a continuous scale suggests that the lowest possible mean on those three tests necessary to get an A is 89.5 , which indicates that David's mean grade of 89.67 is enough to earn an A. Depending on how grades are assigned in your class, this information may be very helpful later on when talking to your instructor about grades.

## Check Yourself!

For a test measured on a scale of 10 to 100 in 10-point intervals, what is the value of the real limit? What, then, are the real limits ( $L L$ and UL) for a score of 70 ?

Real limits will be important in calculating medians and percentile ranks from tabulated data and for constructing histograms for continuous data. You will learn about these in the next few modules.

## Practice

11. What are the real limits of the following scores?

| Score | Size of Each Scale <br> Interval | Real Limits |
| :--- | :--- | :--- |
| a. $\mathrm{IQ}=116$ | 1 point | and |
| b. Height $=66.5$ | $1 / 2 \mathrm{in}$. | and |
| c. Age $=20$ | 10 years | and |
| d. Driving speed $=60$ | 5 mph | and |
| e. Olympic performance $=9.7$ | $1 / 10$ point | and |

(Continued)
12. What are the real limits of the following scores?

|  | Size of Each Scale <br> Interval | Real Limits |
| :--- | :--- | :--- |
| Score | $1 / 10$ point |  |
| a. GPA $=3.2$ | 1 point | and |
| b. Test score $=83$ | 10 miles | and |
| c. Miles to work $=20$ | $1 / 4 \mathrm{ft}$ | and |
| d. Feet of snow $=3.25$ | Whole size | and |
| e. Shoe size $=11$ | and |  |

13. What are the real limits of the following scores?

|  | Size of Each Scale <br> Interval | Real Limits |
| :--- | :--- | :--- |
| Score | $1 / 100$ point | and |
| a. GPA $=3.20$ | $1 / 10$ point |  |
| b. Test score $=83.4$ | 5 miles | and |
| c. Miles to work $=20$ | Whole foot | and |
| d. Feet of snow $=3$ | Half size | and |
| e. Shoe size $=11.0$ | and |  |

14. What are the real limits of the following scores?

|  | Size of Each Scale <br> Interval | Real Limits |
| :--- | :--- | :--- |
| Score | $1 / 2$ point |  |
| a. $\mathrm{IQ}=116.0$ | $1 / 10 \mathrm{in}$. | and |
| b. Height $=66.5$ | Whole years | and |
| c. Age $=20$ | 10 mph | and |
| d. Driving speed $=60$ | $1 / 100$ point | and |
| e. Olympic performance $=9.70$ | and |  |

Visit the study site at edge.sagepub.com/steinberg3e for SPSS datatsets and variable lists.

