

# Strategies for Differentiating Instruction in Grades 3 Through 6

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## 5

### **CHANGING MATHEMATICS INSTRUCTION FOR THE MID-ELEMENTARY GRADES**

As students move into Grades 3 through 6, both the students and the mathematics curricular content change. Of course, students mature throughout school, and it is generally the case that maturity engenders deeper levels of number awareness, number sense, and mathematical skill. Number sense was discussed in the last chapter, and as noted, activities that continue to strengthen number sense are recommended across the elementary and middle school grade levels. The strategies presented in Teaching Tip 5.1 should assist students with number sense in Grades 3 through 6.

#### **Teaching Tip 5.1 Developing Higher-Level Number Sense**

##### **Plan Estimation Experiences**

Estimation is a critical skill for success in higher mathematics, and teachers should teach estimation both directly and indirectly by discussing topics such as “less than” and “more than” with students. Some students are uncomfortable with estimation since students typically strive for the “right” answer. Encourage students to provide estimates within a certain range (e.g., “How many shoes are in class today? Give me a number between 40 and 70,” or “How many students are sitting in a single group in the media center?”).

### **Counting Off in Line**

Students in lower grades frequently line up to go to the lunchroom or elsewhere. Every time students line up, teachers could encourage them to count off, and thus stress number sense (Griffin et al., 2003). As an interesting variation in the upper grades, have students count off when in line using a “quiet voice” but have each person who is a multiple of a certain number (say a multiple of five) say his or her number louder. This makes learning multiplication more interesting since multiples will be frequently used in the classroom.

### **Stress Numbers in Other Subjects**

When encountering a number in a reading story, take a few moments to explore the number. When a group of characters in a story does something together, stop for a moment and say, “I want to get a sense of how many are doing that in this story. Let’s have students in the first row stand up to represent that number.”

### **Emphasize Measurement**

From the early days of kindergarten, teachers should take measurements of objects and discuss them with the class. Teachers may use a short tape measure to measure the length of the teacher’s desk or a student’s desktop. When measuring distances on the floor, teachers may add the element of counting steps (e.g., “How many steps are there in the 10 feet between the front row of student desks and the teacher’s desk?” Have a student walk that distance and count his or her steps. Next, the teacher would measure another floor space of 10 feet in a different area of the classroom, and ask the class how many steps the same student would use to cover that distance.

### **Chart Making Money**

Using charted data in higher grades (which takes only one or two minutes at the beginning of the class period) can encourage students to investigate the use of numbers in a real-world environment. In an economics course, as one example, a teacher might start each student with \$1,000 in make-believe money in the stock market, and have each student pick stocks to buy and to chart stock price changes for. This can be a fun learning activity with real-world significance.

### **Model Enjoyment of Numbers**

Perhaps the most important legacy a teacher can leave with a student is enjoyment of number play. Gurganus (2004) emphasizes the importance of the teacher modeling the enjoyment of numbers and establishing a climate for curiosity in mathematics.

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### **Musical Fraction Squares**

The report of the National Mathematics Advisory Panel (2008) strongly emphasized the need for increased instruction on fractions. In this “musical squares” game, the parts of a fraction can be demonstrated. Begin with four students and four chairs and write “ $4/4$ ” on the dry erase board. Discuss the fact that the chairs represent the numerator and the students represent the denominator. Thus, if the teacher removes one chair while the music plays, there will be  $3/4$  or 3 chairs and 4 students. The teacher can then discuss various fractions with the students and talk about, “What is left when the music ends?” or “What has been taken away?”

Of course, teachers should note that many strategies introduced for primary grades are equally applicable in the higher elementary grades, just as some strategies introduced below will be applicable in the middle schools mathematics classes, as discussed in the following chapter. Veteran teachers are, in reality, masters at adapting strategies, or instructional ideas, to fit their grade level and curriculum, as students’ maturity increases across the grade levels.

In addition to increased student maturity in Grades 3 through 6, instruction in mathematics is also impacted by at least two changes in the mathematics curriculum. After Grade 3 or 4, mathematics becomes both more abstract and more complex. For this reason, some students who succeeded in mathematics in the primary grades may begin to experience difficulties when complex concepts such as fractions, decimals, or two-step word problems are introduced. Further the demands of the mathematics curriculum change somewhat around Grade 3. As noted previously, most instructor’s manuals in mathematics do not provide many representational examples of math problems in Grades 4 through 6, and as the grade levels increase, there is an increased emphasis on problem solving.

*As students move into Grades 4 through 6, both the students and the mathematics curricular content change.*

The Standards for Mathematics Practice associated with the Common Core (see Chapter 1) suggest that students at every level should be provided many opportunities to develop skills in mathematical modeling, and while it is relatively easy to make that statement, such mandates become more of a challenge when a fourth-grade math teacher is confronted with a number of students in the class who have not quite mastered basic operations, or fractions, or one-step word problems. Many students in the middle elementary grades demonstrate

increasing difficulties in mathematics, on work involving simple or complex operations such as multiplication, division, or fractions, and thus, teachers across the mid-elementary grade levels can benefit from tactics for teaching such basic math skills as the times tables, or operations with fractions, and so on.

For that reason, the use of teaching aids such as addition, multiplication, or division math fact charts should be encouraged not only in the primary grades when the times tables are introduced, but in the later and elementary grades as well (Foegen, 2008). Of course, teachers should also make every effort to assist students in reaching automaticity, since automaticity is required in order to succeed in later mathematics courses (National Mathematics Advisory Panel, 2008).

In addition to use of fact charts, there are a number of relatively simple, yet innovative instructional modification options that should be used in the elementary math class (Foegen, 2008; Jitendra, 2002; Joseph & Hunter, 2001). I recommend that teachers implement most of these modifications, presented in Box 5.1, as ideas that will assist struggling students in becoming more comfortable with mathematics.

### **BOX 5.1 Modifications for the Differentiated Math Class**

- Structure new instruction for mathematics concepts as games between teams or as peer buddy activities, using either traditional game boards or computer-based gaming technology. This helps involve students with interpersonal learning styles in the activity.
- Stress “think alouds” to demonstrate how to think through a problem. This provides opportunities for students to communicate on mathematics with each other, as stressed in the Common Core Standards for Practice, (Box 1.1, Standard 3). Teachers should always follow up both correct answers and incorrect answers from students with a question about how the student arrived at that answer. As a modification of this idea, have students consult with a group of “peer colleagues” prior to answering the question, and have other students suggest alternate ways to get the answer.
- Use cue cards for students involved in various types of math problems. The cue cards should present practical steps in problem implementation, and students should be taught how to discriminate when to use which set of cues. Post these around the room for immediate access.
- Use graph paper to assist in lining up numbers and visualizing concepts.
- Always have many counters or pictorial representations available in the classroom, and challenge students who get the correct answer to represent that problem solution for others in the class using these materials.

With these simple teaching modifications as a backdrop in the general education math class, this chapter presents an array of additional strategies for differentiating mathematics instruction in the middle elementary grades, with an emphasis on constructivist instruction to facilitate deeper understandings of mathematical concepts, as envisioned within the Common Core. Also, several more technology-based instructional techniques will be described, in order to foster collaborative work in mathematics, as well as the development of 21st century communication skills among all students as they, working together, discuss mathematical concepts. Finally, a variety of scaffolded instructional strategies are presented later in the chapter, since scaffolded instruction is a very effective platform for differentiating instruction for all students in the math classes.

Of course, readers should note that there is much overlap in instructional techniques across the primary, elementary, and middle grades, so many of the instructional techniques previously discussed (e.g., CRA instruction, technology-based teaching ideas) should be used in these mid-elementary grades as well. However, at least as early as the mid-elementary grades, instruction should focus more on problem solving, in an effort to develop deep, broad knowledge and understanding of mathematics.

## **CONSTRUCTIVIST THEORY AND CONCEPTUAL MATHEMATICS**

Embedded within the Common Core Mathematics Standards, as well as preceding iterations of standards for mathematics, is a strong emphasis on developing deep understanding of mathematics concepts and problem-solving skills (Garellick, 2012; National Mathematics Advisory Panel, 2008). Many instructional experts have suggested that “constructivist theory” might be the most appropriate perspective from which to develop deep conceptual understanding in math across the grade levels (Grobeck, 1999; E. D. Jones, Wilson, & Bhojwani, 1997; Woodward & Montague, 2002). Constructivist theory calls for students to be perceived as learners who may be immature in their mathematical understandings, but who can develop deeper knowledge in math as long as they

Have mastered the prerequisite skills for a particular problem, and are supported by the teacher and curriculum as they “construct” meaning or further understandings of the mathematics problems under study.

In the constructivist view, teachers are not information providers (demonstrating problems on the board), but rather are facilitators who provide opportunities to learn along with appropriate supports to assist students in developing their growing understanding of various mathematical skills. In that sense, mathematics teachers should function more as “coaches” rather than “modelers” of problem-solving techniques. Further, as students mature in their cognitive understandings of mathematics concepts, the teacher should withdraw the supports and allow students to work more independently.

According to this perspective, in order to accomplish effective instruction teachers need to develop an array of instructional skills. First, teachers must understand what supports a student may need based on the student’s current mathematics skills and their understanding of the problem. Next, for a given student, the teacher must apply the specific type and level of support needed, and at a later point withdraw it, as student comprehension increases. Closely aligned with this perspective is the notion of “guiding” the cognitive understandings of students as they develop in their mathematical experiences (Alsup, 2003). I should note that I have chosen to present this constructivist concept in the discussion of mathematics instruction for Grades 3 through 6, but this construct is every bit as relevant for lower grades, as well as middle school grades.

Teachers do this “guiding” by listening to a student’s solution to a problem and then asking pointed topical questions that are designed to guide students cognitively through the problem-solution process. Thus, students are responsible for using the information they have learned to solve the problem, and the students are exposed to the mystery and the fun of problem solving in math. Students may use manipulatives or models as described in previous chapters in order to solve the problem, but the emphasis will be on moving students past the necessity for concrete or representational models and into a deeper level of conceptual understanding. In that sense, the constructivist perspective fits very nicely with the main emphasis of the Common Core Math Standards on development of deep conceptual understand of mathematics processes.

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### **Scaffolded Instruction**

Arising from this constructivist perspective, teachers began to use the term “scaffolded instruction.” Scaffolding is an instructional technique

used in many different subjects, including mathematics, and originally represented a procedure whereby teachers strategically guide students in the learning process by questioning students in order to assist in building their understanding of a mathematics problem (Woodward & Montague, 2002), and then withdrawing that guidance as the student's skills increased. However, it was soon realized that many instructional procedures other than individual teacher tutoring can support student learning by assisting the student to understand content (e.g., graphic organizers, pictures). Thus, scaffolded instruction may best be understood as a sequence of prompted content, materials, and teacher or peer support to facilitate learning (Grobecker, 1999; Karp & Voltz, 2000). In scaffolded instruction, the emphasis is placed on a teacher's assisting the student in the learning process with individual prompting and guidance that is tailored to the specific needs of the individual student and offers just enough support (i.e., a scaffold) for the student in a new task (Karp & Voltz, 2000). Further, that support should gradually be withdrawn, allowing the student to eventually "own" the task performance. The student, then, is initially considered an apprentice, and soon to be an expert, in the learning effort.

### **Constructivist Perspectives and 21st Century Technology**

In the collaborative instructional world of the 21st century classroom, the constructivist perspective is perhaps even more relevant than in more traditional classes. As students use various apps in mathematics, different math websites, and various Web 2.0 tools (e.g., blogs, wikis, Google docs, and collaborative, cloud-based assignments, etc.), they will be using technology as scaffolds to support their understanding. Further, these 21st century tools allow students to collaborate in completing mathematics problems in ways that could not be envisioned even 20 years ago. In this collaborative context, students often assist other students in their efforts to "construct" understanding of various math examples, many times using these 21st century tech tools. From the constructivist perspective, these student peer-buddies are using technology to do the instructional "guiding," and for many students who might fear mathematics somewhat, this can be much less threatening than traditional instruction from the teacher, even if that instruction is delivered in a one-on-one teacher tutoring session.

In that sense, as this book is written in 2013, constructivist theory is becoming even more influential in education, because it does stress deeper conceptual understanding and applications of math, and cross-pollinates nicely with the ongoing development of tech-based teaching strategies. Math teachers should certainly avail themselves of these powerful, new constructivist, tech-based teaching tools.

## TECH-BASED TOOLS FOR DIFFERENTIATING ELEMENTARY MATHEMATICS

*In the collaborative instructional world of the 21st century classroom, the constructivist perspective is perhaps even more relevant than in more traditional classes.*

The last chapter presented two tech-based instructional strategies—computer-based gaming and use of avatars to personalize mathematics—to make this curricular area less threatening and more engaging. However, 21st century teaching tools can go much further than this, and can actually make mathematics a collaborative, creative activity for individual students or for groups of students. Using recently developed tech-based teaching tools that are generally referred to as Web 2.0 tools, students can grapple with mathematics content collaboratively, in a highly engaging, social manner, and assignments based on these tools tend to transform the mathematics class into a much more exciting learning experience. In today’s classroom, students are not merely consumers of information, but actually create content that can then be shared worldwide, and studied by others, using technology. Thus, mathematics is transformed from “learn these procedures and formulas” to “creatively engage with your peers in the class and online, to devise a workable mathematics solution for this problem.”

Both class blogs and wikis are presented in this chapter, as the initial tools teachers might use in fostering such collaboration. Even for teachers with little background using instructional technologies, these tools can be easily mastered. These are 21st century teaching tools that can enhance differentiated instruction, and generally enrich the mathematics experience for all students.

### **Blogs for Mathematics Instruction**

Students in the mid-elementary grades typically begin to socialize much more with their peers than in earlier years, and many technology tools can use this growing desire for socialization and harness that desire as an instructional tool. For example, many teachers today—math teachers among them—are using blogs to help students collaborate in mastering curricular content. Specifically, this tech tool helps mathematics teachers address Common Core Mathematical Practice Standard Number 3, which emphasizes the construction and communication of viable mathematics solutions.

A blog is a collaboratively developed, online journal in which posted information from both teachers and students is arranged and archived in reverse chronological order (W. M. Ferriter & Garry, 2010). Using a blog,

teachers and students can work individually or together to create a solution to a mathematics problem or recommend information to their peers relative to that problem, by creating online links to other material such as videos or model problems. All blog entries are listed, in order, so teachers can follow which students added specific comments. Students can collaboratively work on a math problem, with each student in the group completing steps in the problem in turn. Blogs provide the opportunity for connectivity in that the students can leave comments on the work from other students. Finally, when teachers create classroom blogs, students are able to interact with the mathematics content long after the school day has ended.

I recommend that 21st century mathematics teachers use blogs professionally, in at least two distinct ways: as a professional learning tool and as a class instructional tool to foster student excitement about mathematics. First, blogging has become an excellent way to share information and teaching

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ideas professionally, and many blogs for math teachers are available. Teachers may wish to check some of the most popular general topic blogs for teachers ([edudemic.com/2011/12/teacher-blogs/](http://edudemic.com/2011/12/teacher-blogs/)). However, several mathematics-related blogs are also good starting points for teachers interested in mathematics instructional

ideas, as presented in Box 5.2. From blogs such as these, teachers can pick up an amazing array of teaching ideas, with relatively little reading time.

### BOX 5.2 BLOGS FOR TEACHING IDEAS IN MATHEMATICS

*The Number Warrior*—This blog, written by an Arizona high school math teacher, offers a mix of riddles, observations, and videos on higher-level topics in mathematics ([numberwarrior.wordpress.com](http://numberwarrior.wordpress.com)). Various games and cooperative learning ideas are included here as well, and all mathematics teachers are invited to join the discussions.

*Math for Primates*—This blog includes a series of podcasts on math that present various mathematics topics in an entertaining fashion ([www.mathforprimates.com/](http://www.mathforprimates.com/)). This site can be used to glean teaching ideas, as well as by more advanced math students. Recent posts explore the idea of infinity, as one example.

*Continuous Everywhere but Differentiable Nowhere*—This blog is written by a math teacher in New York, and describes various questions faced by mathematics teachers ([samjshah.com/](http://samjshah.com/)). I found several discussions of how this teacher might manage his class (e.g., do one problem initially, then assign peer buddies to complete a problem), and it can be informative for math teachers to see another math teacher “think through” how to structure the math class.

*Let's Play Math*—This blog presents a great deal of content on teaching mathematics, including various games and teaching suggestions ([letsplaymath.net/](http://letsplaymath.net/)).

*I Speak Math*—This blog is written by a middle school math teacher in North Carolina and presents many creative teaching ideas such as creation of a video on operations with fractions, or hold a free-throw competition on decimals ([ispeakmath.wordpress.com](http://ispeakmath.wordpress.com)).

*Keeping Math Simple*—This blog focuses on math pedagogy, and provides many instructional ideas ([math4teaching.com/](http://math4teaching.com/)). The blog author/creator, Erlina Ronda, seeks to narrow the gap between theory and practice.

*Math Teacher Wiki*—This is an open wiki that teachers can share ideas on for math instruction ([msmathwiki.pbworks.com/w/page/27071816/Math-Teacher-Wiki](http://msmathwiki.pbworks.com/w/page/27071816/Math-Teacher-Wiki)).

When considering a blog for student use, teachers should realize the value of social learning in the 21st century classroom. The extreme popularity of the many social networking sites available today demonstrates students' desire to be socially connected, and math teachers should certainly tap into that desire for social interaction. A blog is the simplest way to do that.

Utilizing class blogs fills that desire for socially mediated learning by offering high levels of student-to-student interaction focused on the mathematics content (Richtel, 2012). Students are more likely to enjoy mathematics taught in this fashion (Richtel, 2012), and they are much more motivated to complete mathematics homework when it involves collaborative blog postings on the problem or problems under study.

To use a blog as a classroom teaching tool, teachers simply write a note (called a “post”) about a particular topic to begin the blog, and then encourage students to respond to that post, with a post of their own. The teacher's initial post might be a general question on a mathematics concept, or an actual math problem for students to discuss. Here is a sample blog post teachers may use when teaching rounding from the Common Core Math Standards in Grade 3:

*Students are much more motivated to complete mathematics homework when it involves collaborative blog postings on the problem or problems under study.*

Use place value understanding to round the whole numbers to the nearest 10 or 100. (3.NBT.1)

*Use place value understanding to round the whole numbers to the nearest 10 or 100. (3.NBT.1)*

Teacher: *Who has an idea about how to round off the number 76? Can anyone explain how we do that?*

Student posts might include a variety of ideas such as:

Laquisha: *I think that the six in the one's place is higher than five so we need to round up. That means the answer is 80.*

Billy: *It would be 70. Right?*

Students can then post their responses to the teacher, or respond to other students' comments. Here are several examples.

Stacy: *Laquisha has it right. I think we need to go up to 80, because you go up if the number in the one's place is five or higher.*

Adam: *Laquisha and Stacy are correct. The answer is 80.*

As this simple example indicates, even a general blog discussion tends to be quite collaborative in nature. Further, students, with just a bit of coaching, can be taught to focus on the correct ideas, and ignore the incorrect ones, such as Billy's idea above. Alternatively, students may be taught to gently coach other students on the blog when an error is made. In these ways, blogs can be nonthreatening, and yet highly interactive. Further, blogs are highly accessible as students can log onto blogs via their personal computers, mobile devices, or any other technology tool that has Internet capabilities, either at school or at home. On some blog platforms, students even have the option to "follow" blogs, so that the students would be notified via e-mail whenever there is a change on the blog itself.

Blogs have been used by various educators for many years now, and do tend to increase student engagement and collaborative learning. However, other teachers have not employed this tool. In some cases, the school was not Wi-Fi equipped, or few computers were available for students. Still, this is a tech tool that mathematics teachers should begin to utilize in the class, since it is a 21st century communication skill, and provides many collaboration options. Finally, blogs are fairly easy to set up, even for the uninitiated teacher who has not used this tech tool before. The steps in Teaching Tip 5.2 should help in that regard.

### Teaching Tip 5.2 Setting Up a Math Instructional Blog

#### Select a Blog Hosting Site

First, teachers must select a blogging host website for their blog. Some sites are available at no cost, but these tend to be funded by advertising, while others are ad-free. Teachers should also consider the levels of privacy (who can participate in or view the blog—students only or students and parents?), as well as student security. A number of hosting sites are available including:

[www.classblogmeister.com](http://www.classblogmeister.com)—This is a free website designed specifically for teachers and classroom use. All articles and comments are sent to the teacher for approval prior to being published, which helps the teacher ensure acceptable behavior on the blog. Teachers can also ensure that the site is password protected.

[www.21classes.com/](http://www.21classes.com/)—This is a blog host with several options for teachers. A single teacher blog is free for teachers and provides the teacher with a central dashboard for controlling accounts and comments. This free option allows for uploads of videos and images. The blog can be made public or private for the classroom. Regardless of the setting, the teacher has control over which posts are approved and even if certain posts are only approved for certain groups. 21 Classes also offers fee-based programs where students have their own personal blogs monitored by the teacher.

[edublogs.org/](http://edublogs.org/)—Edublog is another host created entirely for educational use. It allows teachers to make their blogs private or public. Edublogs boasts an adult-content-free site, making it safe for students to browse through the site. There are several options for teachers when using Edublogs. The free version allows students to create their own blog without having an e-mail address. The site recommends that a single teacher use the Pro version, which is \$3.33 a month for 50 student blogs as well. This Pro version has more options for teacher monitoring.

[education.weebly.com/](http://education.weebly.com/)—Weebly is a free service for teachers. Due to the emphasis on education, no advertising appears on the blogs. The service is completely free for up to 40 student accounts. The host supports picture, video, document, and photo gallery uploads. All websites (either teacher or student created) have the option to be password protected, offering higher levels of security for parents and teachers. Weebly boasts an easy drag-and-drop editor, making the creation of any site easy.

### **Consider the Blog Audience**

Generally, students are more motivated to actively participate in mathematics activities on the blog if the blog audience is larger; a bigger audience fosters excitement about the blog (Richtel, 2012). Blogs that require a student login and password can be secure while allowing a wide audience. Making blogs available to the school and parent community encourages students to think about their audience and how to engage them, and fosters student excitement about using the blog. Ultimately, this will increase students' motivation and commitment to the mathematics discussion.

### **Follow Blog Host Instructions**

Finally, teachers should follow the hosting and set-up instructions for their particular blog. Again, anyone who currently uses e-mail can set up a blog, but instructions are different on the various hosting sites. I generally recommend that teachers look at one or two of the blog hosts, and watch the tutorial videos

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on that website, in order to learn the tips and tricks of that particular blogging host.

### **Create the Initial Post**

The initial blog post should be something that will engage students' curiosity and motivate them to want to participate. Teachers might initially ask a general question, and, somewhat later, post a specific mathematics problem for students to consider. Links to outside documents, model problems, video demonstrations, pictures, or audio clips can also be included in the blog.

Blogs provide many differentiated instructional options in the elementary mathematics class. For example, blogs allow teachers to make meaningful, content-rich assignments for some students, while still allowing them to provide direct instruction. As one example, imagine a fourth-grade mathematics teacher, Ms. Chang, who is teaching measurement and focusing on the Common Core Measurement and Data Standard (4.MD.3);

*Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

In most fourth-grade classes, both the academic skill level and the learning preferences vary considerably. Thus, Ms. Chang would probably have several students in her class that require extra instruction from her on that standard, while others may have mastered that work and need more challenging work. In order to differentiate instruction, Ms. Chang could use her class blog to make a collaborative, creative assignment. She could require the advanced students to work on the blog, reviewing a link to a video about areas of multishaped buildings (e.g., instances in which students must calculate areas for two or more building sections, and then add those areas together for the total floor area). Those students would view that video, and then post questions and comments on the blog about that type of calculation. Those students might then be required to make up several such area calculation problems, based on rooms within the school. At the same time, Ms. Chang could provide direct instruction for the other group of students. In this manner, Ms. Chang would be able to

differentiate the content through the use of the interactive class blog without embarrassing any students in a class setting.

Blogs can also help differentiate in other ways. Because teachers and students can link any digital document, audio, and video clips to a class blog, many learning options are offered for students with different learning styles that may not be offered in the traditional classroom setting. Material presented in class can be differentiated on the blog by including various presentations of the same math content. For students who may need a more visual representation of the math example, such an example could be posted by the teacher. In contrast, students needing auditory explanations may be referred to a podcast on the content, with a link to that podcast posted within the class blog. In fact, with multiple materials linked to the class blog, students can search for the media selections that best help them understand the math content in a clear manner.

*Blogs make differentiating instruction easier by providing ways to differentiate the class activities.*

In this sense, class blogs can be used as a time saver. Teachers often note that their actual instructional time in the class is quite limited, and as the example above demonstrates, well-developed class blog assignments can provide teachers with an assignment option for one or more groups of students while leaving the teacher free to work with others.

Blogs can also provide limited options for student creativity. For example, when studying various shapes in mathematics, teachers may have students use their mobile phones to take a picture and upload examples of the shapes they see in the community or in their home. Further, students may be required to post some brief comments on the blog about their shape and why they found it interesting.

### **Wikis in the Mathematics Class**

While blogs do facilitate class interaction, as noted previously, a more effective tech tool for increasing collaboration and joint problem solving in mathematics is the wiki (Watters, 2011b; Wetzel, 2012). A class wiki is similar to a blog in that all students can post contributions to the wiki, and this results in more options for true collaboration on mathematics projects. However, wikis involve much more functionality, and more instructional options than merely class blogs. In fact, a wiki is, essentially, an editable website for the class, and may include class blogs, as well as many other 21st century instructional tools. Wikis can be used as instructional tools in the mathematics class, or they may be set

*A wiki is an editable website, usually with limited access, that allows students to collaboratively create content and post their work in the form of digital files.*

up as communication tools, to let parents, other teachers, administrators, or anyone else review the work done in their class. Wikis usually involve limited access, in order to protect student identities, but they do allow students to collaboratively create and post

written work, or share digital files, such as digital photos or even digital video projects (Watters, 2011b; Wetzel, 2012).

Wikis, like blogs, have been used in some classrooms since the late 1990s (Richardson, 2010; Watters, 2011b). Some teachers use the class wiki as a combination of a unit syllabus and instructional activities assignments page for math students. Today, teachers can do virtually all of their instruction based within a class wiki, because wikis are so very versatile and can include any type of assignment a teacher might imagine. Box 5.3 presents a number of ideas for using wikis in the mathematics class.

### BOX 5.3 INSTRUCTIONAL IDEAS FOR WIKIS IN THE MATHEMATICS CLASS

All of the following teaching ideas for wikis could also be represented by pages within a class wiki. Teachers would set up the parameters for these teaching ideas on a locked wiki page, and then provide unlocked wiki pages for students to work with.

*Problem of the Day*—Wikis can present a teacher-developed problem of the day or a problem of the week, and students can work, individually or in teams, on that problem with reinforcement offered for the first team to solve it.

*Create an Example*—Wikis can provide a location for students to upload interesting images representing mathematics concepts that they take with a digital camera (typically a parent's smartphone) in the local community. Students should be required to write a brief paragraph about the image, and its importance in the mathematics unit under study.

*You Write a Problem*—Working in teams, students can be required to create a sample mathematics problem based on the topic under study, and upload that to the wiki. Others would then critique that problem and perhaps solve it.

*My Math Problems Today*—Students might individually present a mathematics problems that they encountered in their daily lives. Unlike the "You Write a Problem," this assignment involves a mathematics problem of any type and is not related to the unit under study.

*My Math Vocabulary*—On this page, the teacher should input all math terms for that specific unit of instruction, and perhaps any review terms from the

previous unit. Students would then be expected to define and provide examples for those terms.

*Around the World Collaboration*—Using a wiki, teachers can help students work with other students anywhere in the world on mathematics problems. For example, students in Utah might share pictures of a mountain valley and a calculation of the rough area of that valley, while students along coastal North Carolina might provide a picture of a sound or river basin, and likewise present an example of the area of that geographic feature. The website iEARN ([www.iearn.org](http://www.iearn.org)) is dedicated to helping teachers around the world connect their classes for this type of collaboration.

As described earlier, blogs preserve students' and teachers' posted comments separately, in chronological order, and while the blog can facilitate some collaboration, students generally cannot work directly on the same mathematics problem at the same time. However, by using a wiki, teachers can encourage a much richer student collaboration, since in the wiki students can actually edit the work of others, rather than merely comment on it. Thus, students can collaboratively solve problems while working within the same digital file in the wiki. The most recognized example of a worldwide wiki is, of course, Wikipedia, a free, online encyclopedia that anyone can contribute to or edit ([www.wikipedia.org](http://www.wikipedia.org)).

These collaborative and creative options make a wiki a true Web 2.0 tool, since creative collaborative options can be explored in wikis that have not been possible previously. Thus, when a group is working as a small group on a specific mathematics problem, students can and will correct each other's mistakes. This functionality makes wikis an excellent tech tool for increasing the types of collaboration and social learning that students today enjoy. Also, like blogs, most wikis allows teachers to track every posted entry to see who is making entries and who in the class is not, and even in collaborative work, that feature is quite useful for following individual student contributions.

Wikis can also be used to encourage students to publish their own work contributions in a mathematics problem-solving scenario (Wetzel, 2012). Students will quickly get used to editing everyone else's content, and over time, this leads to an online, collaborative community of math learners who serve as a resource for each other. In that sense, wikis will move your students toward increased participation and increased enjoyment of math. Again, students

*With a wiki, teachers can encourage a much richer student collaboration, since in the wiki students can actually edit the work of others collaboratively, while working within the same digital file.*

are demonstrating by their out-of-school actions that they love social networking, and by using that motivation for increased social exchange in the context of mathematics instruction, teachers can use a class wiki to encourage students to participate more fully in the classroom and while completing math homework.

Further, the skills developed within the wiki truly represent 21st century skills. In developing written or video content for the mathematics wiki, students learn how to work together, sort through information, evaluate information using various sources, create newly synthesized information, and make contributions to the content already on the wiki (Bender & Waller, 2013; Richardson, 2010). These are the skills that will be demanded by the 21st century workplace, and merely using a wiki teaches these skills.

Wikis serve one instructional function better than any other teaching tool: Wikis are excellent for teaching mathematics vocabulary! From the mid-elementary grades and up, mathematics curricula involve developing vocabulary, and many teachers spend some time on math vocabulary in each unit of instruction. In contrast, using a class wiki in math can save that time!

Within the wiki, teachers can merely list the vocabulary terms for the instructional unit and have students define the terms and provide exam-

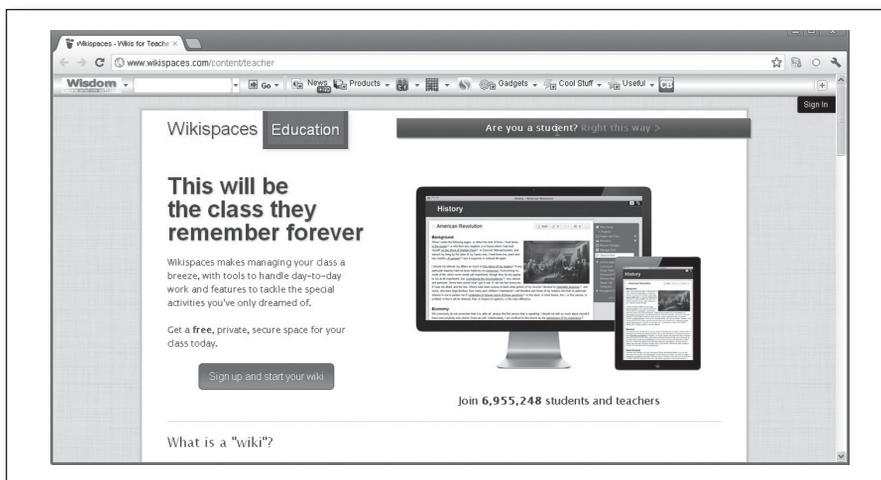
*Wikis serve one function for teachers, perhaps better than any other teaching tool: Wikis are excellent for teaching mathematics vocabulary.*

ples in the wiki. Students can pick up the definitions for each term as they work through the wiki, while checking the work of their peers. Teachers should merely list vocabulary terms on an “unlocked” (i.e., editable) wiki page, and have the students define those terms. By requiring that such work be completed on the class

wiki, the teacher saves valuable class time. Students still get the content, however, in a manner similar to their use of Wikipedia. Most students enjoy this type of activity and will learn vocabulary terms based on this activity, without the teacher having to take additional class time to teach the vocabulary for mathematics.

As schools today are striving to make Wi-Fi and computers or tablets (e.g., iPads) available, many teachers are considering how to use this Internet connectivity in the math class. A wiki is one of the best ways to do so. In fact, teachers who have never used a wiki previously can set up a wiki for their class in approximately 30 to 45 minutes. Further, the Internet provides many sites that will assist teachers in developing and using wikis. I typically suggest that teachers use the Wikispaces website ([www.wikispaces.com](http://www.wikispaces.com)) for setting up a class wiki, since that service is free for teachers (see Figure 5.1).

Figure 5.1 Wikispaces Website



Source: Wikispaces © 2012 Tangient LLC

Today seven million teachers and students are already using Wikispaces for their class wiki. In order to help you learn how to use this wiki, we recommend that you review brief videos from the Wikispaces homepage ([www.wikispaces.com/content/wiki-tour](http://www.wikispaces.com/content/wiki-tour)). While the actual steps in setting up a wiki for a math class vary depending on the website you choose, the general steps in Teaching Tip 5.3 should help you get started on your class wiki.

### Teaching Tip 5.3 Creating a Math Wiki for Your Class

#### Select a Wiki Name and Password

Begin by going to the Wikispaces website ([www.wikispaces.com/site/for/teachers](http://www.wikispaces.com/site/for/teachers)). There you may select the option to set up a free wiki (toward the lower right of the homepage). Next you will be asked to select a username, password, and a Wikispaces name.

#### Select Your Security Level

Student security is critical as students begin to use the Internet, so I suggest that teachers create a private wiki, which only class members and school administrators can view or edit. Later you may wish to make the wiki available to parents for viewing only (not editing). The private wiki option is free for educators.

*(Continued)*

(Continued)

### **Create a Wiki Homepage**

As the first page within the wiki, you should create a homepage that includes a brief introduction to the class and other basic information. Teachers first create a title for the wiki at the top of the homepage, using the “Heading” function at the top of the page (select “Level One Heading.” That will bold the heading and increase its size.

Next, I suggest that a title page should have an opening or introductory paragraph. This should be a paragraph-long description of the content and purpose of the wiki. You should phrase this paragraph as an “interest grabbing” paragraph, and using questions is very appropriate.

### **Add Videos or Digital Photos**

The goal of the wiki homepage is to excite the interest of the students, so teachers should add a set of interesting photos or videos on mathematics related to the topics that will be studied. You may also add another heading such as “Recommended Videos,” or “Problems you will soon learn about!” Below that heading you will put links to videos you select that can grab students’ attention, or examples of problems to be studied. You can find interesting math videos on many of the websites previously mentioned or on YouTube, TeacherTube, PBS.org, or the Discovery Channel. Generally you should select shorter rather than longer video segments (3- to 10-minute video examples).

### **Add a Navigation Option**

Next, you should select a navigation option, since you will add many more pages to your wiki. The navigation option lets students know how to get from one page to the next. In the edit bar at the top of the page is a button called “wedges.” Click that, and a list of options to add to your wiki will open, one of which is “add the navigation tool.” When you click that, a navigation tool will be added at the bottom of the page you are on that will allow you or your students to navigate to other wiki pages.

### **Create Locked and Unlocked Wiki Pages**

Now you are nearly finished with the homepage, and you now need to lock up the content that you don’t want students to edit. In most cases, teachers don’t want the students (or anyone else) to edit the content on the homepage, so that page of content needs to be locked down. While many wiki pages are available for students to edit, the homepage generally should not be.

To lock a page in a wiki, move the cursor to the top right of the edit bar, over the series of dots, and click there. Then you are presented with options, one of which is “Lock” this page. Click that once you are completely done with your homepage. You will need to repeat that “lock” for each page you want locked

down. As creator of the class wiki, you will always have the option of unlocking any page for your own edits. However, you will need to lock it down again at the end of that process.

### **Create Other Wiki Pages**

Next, you will need to create additional wiki pages on the math content that students can edit. These “unlocked” pages allow students to make creative contributions to the wiki. One of these should be a vocabulary page, discussed previously in text, and Box 5.3 presents a series of instructional ideas, each of which could be a separate page in your class wiki.

### **Adjust the Look of Your Wiki**

There are a variety of options to change the color of items or the background color of your class wiki, and you should play with those options over time. Teachers can use the “edit this page” tab in the tool bar at the top of the page to manipulate text, change fonts, and adjust spacing for each page. While this is not really essential, a nicer looking wiki can be more engaging for students.

### **Invite Students and Parents to Join Your Class Wiki**

Once your wiki is ready, teachers must invite students to join the wiki using the “User Creator feature.” In some cases, teachers can sign their students up directly, and that is preferable to the invitation process. After using two or three wikis for the initial instructional units in your math class, you can consider inviting the parents to join the wiki as observers. This can foster a great deal of good will for you, and it is almost guaranteed to improve the parents’ perception of your instructional skills.

I always recommend, as a student safety option, that at least one other educator be included on a class wiki. This might include administrators, department chairpersons, or other school leaders, who would not really follow the wiki, but might access it from time to time. This can serve as a useful oversight function if anyone ever raises a question about any of the content on the wiki.

Differentiating instruction using a class wiki is relatively easy, because any differentiation option you can use in the general education classroom is likewise an option for the wiki. However, since wikis allow students to participate in their learning through actual creation of content for the wiki, students will tend to segregate themselves somewhat, based on their learning styles, preferences, and strengths, as they complete activities collaboratively within the wiki framework. In that sense, differentiated instruction begins to take place almost without teacher input, when instruction is based in a class wiki.

For example, interpersonal learners may be more inclined to work together on an online of a geometry problem, whereas students who are more

*Since wikis allow students to participate in their learning through actual creation of content for the wiki, students will tend to segregate themselves somewhat, based on their learning styles, preferences, and strengths.*

inclined to movement-based learning may wish to work together and develop a “walk-through map” of the geometric problem. Either of those could be used to generate a digital video-tape that could then be uploaded to the class wiki. As this example illustrates, students will have many options to exercise their learning strengths when wikis are used within the math class.

### **Calculators in Mathematics: A Running Debate**

For years, teachers of mathematics have debated and even decried the use of calculators in mathematics (Fahsl, 2007; National Mathematics Advisory Panel, 2008). Some teachers in elementary, middle, and high school allow or encourage the use of calculators, whereas others discourage such use, based on concerns that calculators may ultimately impair a student’s instant recall of math facts. Some teachers suspect that use of calculators may even impair the student’s deeper understanding of mathematics.

Clearly, as students progress into higher mathematics including problem solving, their knowledge of math facts becomes critically important, and most students do ultimately reach some level of automaticity (instant memory) for math facts. In later mathematics, when the emphasis in the mathematics lessons may have shifted away from calculation and math facts into more complex problem solving, such automaticity greatly facilitates success in math. However, many students with learning disabilities and others who are struggling in math may not have achieved automaticity in basic math facts (Foegen, 2008; L. S. Fuchs, Fuchs, Powell et al., 2008), and the use of a calculator for certain functions in mathematics may be desirable for struggling students.

The Common Core Mathematics Standards, and in particular the Standards for Mathematical Practice, allow for the use of calculators (see Standard 5, in Box 1.1 of this book). However, this should not be taken to mean that students should not learn their basic math facts. The National Mathematics Advisory Panel (2008) reviewed research on the use of calculators and found little support either for or against the use of calculators in the math curriculum, though the report did note that use of calculators in lower grades was a serious concern among algebra teachers. The report further cautioned that to the degree that use of calculators impedes development of automaticity with math facts, the student’s fluency in computation would be adversely affected.

With these concerns noted, caution is advised concerning the use of calculators in mathematics, and teachers who allow the use of calculators should judiciously consider appropriate use of these technology supports. Fahsl (2007) suggested that students be allowed to use calculators on only certain types of tasks (e.g., on checking work that has already been completed or solving complex problems). However, if the aim of a particular mathematics lesson involves automaticity in math facts, the use of calculators should not be allowed.

## **DIFFERENTIATED INSTRUCTION FOR DEEP CONCEPTUAL UNDERSTANDING**

As noted previously, the cognitivist perspective suggests that students construct their understanding of new math concepts based on previous knowledge and various supports provided by the teacher relative to new content material. In addition to the tech-based strategies above, a number of other strategies are used as scaffolds within this cognitivist perspective to help students construct mathematical understandings in the mid-elementary grades. The strategies presented below exemplify the types of instructional practices recommended today, ranging from graphic instruction for math facts, to various strategies for problem solving in the mid-elementary grades.

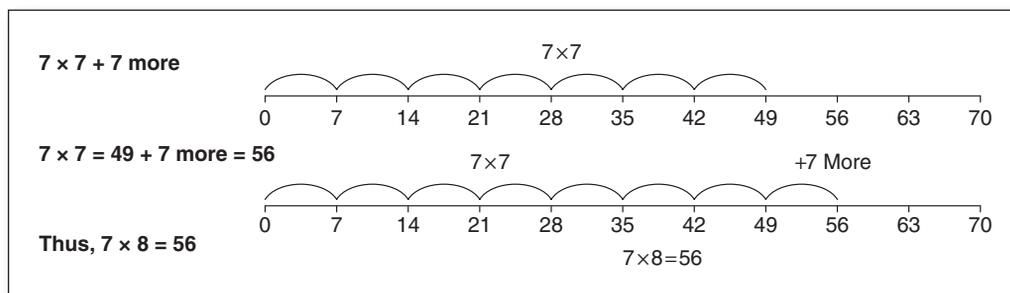
### **Graphic Instruction for Times Tables**

Instead of rote memory for learning the times tables, representational examples or even concrete examples can also help. Woodward (2006) described a graphic example for teaching times tables, based on conceptual and representational manipulation of previously derived facts. Specifically, in teaching the higher times table facts, students might be taught to utilize what they already know and from that derive the correct answer. When a student is confronted with the fact  $7 \times 8 = ?$ , and he or she already knows that  $7 \times 7 = 49$ , the student can derive the necessary math fact by thinking the problem through based on what is already known. The problem of  $7 \times 8 = ?$  may be conceptualized on a number line as demonstrated in Figure 5.2.

### **Pattern Times Tables Instruction**

The Common Core Math Standards clearly emphasize patterns in mathematics (see Math Practices Number 7 in Box 1.1, Chapter 1), and times tables are an excellent vehicle for pattern-based teaching. In fact, teachers have discovered that for many children it is critical to start with

Figure 5.2 Number Line



easy patterns prior to moving into more difficult patterns (Lock, 1996). As an example of a skill progressing from easy to more difficult, consider instructions in times tables (multiplication math facts). Rather than teaching the times tables in order (1 x table; 2 x table; 3 x table; 4 x table, etc.), as is typically done, teachers can teach the multiplication patterns fact from easy to more difficult by teaching the 1 x table and the 2 x table, followed by the 5 x table and the 10 x table. This is easy because students can be taught (or may already know) the one and two levels, and may even be able to count by five or by 10. Thus, the first tables taught tend to be confidence building and easy to memorize.

Next, one might teach the “squares” pattern (i.e.,  $3 \times 3$ ;  $4 \times 4$ ;  $5 \times 5$ ;  $6 \times 6$ , etc.). Then, the teacher might emphasize the commutative property in reversed problems ( $3 \times 4 = 4 \times 3$ ). Next, the teacher might produce a multiplication chart of math facts of all the multiplication tables from one through 10, and use that to illustrate how many facts in the new times tables the student already knows from the lower times tables. Thus, the times tables can be made easier by teaching from easy to more difficult patterns.

### Process Mnemonics for Computation

Another tactic that facilitates differentiated instruction in computation is based on process mnemonics (Higbee, 1987; Manolo, 1991; Manolo, Bunnell, & Stillman, 2000). Mnemonics are acronyms or sentences that may be used for assisting with memory tasks, and this tactic has been used for many decades in reading instruction. For example, simple process mnemonics have been used as reminders of processes in a variety of language arts tasks over the years (e.g., “i’ before ‘e’ except after ‘c’ and when sounding like an ‘a’ as in neighbor and weigh”). However, little use has been made of process mnemonics in math instruction, until the last several decades (Manolo et al., 2000).

Mnemonics are used in Japan to summarize the organization and the process of problem solving (Manolo, 1991; Manolo et al., 2000). Manolo and his coworkers (2000) hypothesized that since process mnemonics had been shown to be an effective teaching tool for students with normal abilities, it could hold promise for use with students who are struggling in math.

Process mnemonics utilizes representations of constructs in math that assist students with memory. In one research study, Manolo and others (2000) found they could engage students more actively in the mathematics process by presenting numbers as “warriors” and mathematical operations as situational military stories to teach the rules and procedures necessary for numerical addition, subtraction, multiplication, and division of whole numbers and decimals. Several example adaptations of this tactic are presented in Box 5.4.

### BOX 5.4 Process Mnemonics Examples

#### Process Mnemonics for Teaching Decimals

**Subtraction:** The process mnemonic for subtraction of whole numbers and decimal fractions tells students to imagine that the two numbers are different sets of warriors doing battle. The digit on each warrior denotes his strength. Warriors to the left of the decimal point are ranked, and those to the right of the decimal point are not ranked. To fight, the warriors must be lined up ranked against ranked and unranked against unranked in battle—this reminds the students to line up the decimal places prior to subtracting. The top group of warriors in the subtraction problem is designated the defenders and the bottom group is the attackers. The attackers can be recognized because they have their swords drawn (i.e., the minus sign in the problem).

$$\begin{array}{r}
 24.6 \quad \text{defender} \\
 -12.7 \quad \text{attacker} \\
 \hline
 \end{array}$$

When subtracting, if the defender’s strength in any column (i.e., the number in the top row) is less than the attacker’s strength (the number in the same column on the bottom row), the defender has to be increased by ten in order to do battle. The consequence of this is that the next defender has his strength reduced by one. For example,

$$\begin{array}{r}
 24.6 \quad \text{changes into} \quad 23.6 + 10 \\
 - 12.7 \quad \quad \quad \quad \quad - 12.7 \\
 \hline \quad \quad \quad \quad \quad \quad \quad \hline
 \end{array}$$

Students then solve for the answer: 11.9

**Addition:** Addition problems are presented as the warriors getting into a boat (Manolo, 1991; Manolo et al., 2000). The addition sign is the mast and the line

*(Continued)*

(Continued)

below is the boat itself. Students should be taught that the warriors (numbers) to the left of the decimal point are ranked and those to the right are unranked, just as in the subtraction example above. They must always line up the lines of warriors ranked with ranked and unranked with unranked. If there is an unranked warrior lined up in the same column with a blank on the right of the decimal point, then the student should put a zero in that blank space, since a warrior with a rank of zero really doesn't matter and doesn't affect the outcome at all. In that type of situation, the problem changes as follows:

$$\begin{array}{r} 39.4 \\ + 12.45 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 39.40 \\ + 12.45 \\ \hline \end{array}$$

If the numbers have no dots, then students could put dots to the right of the warrior farthest to the right, and then line up the warriors by using the dots. Finally, they would solve the problem.

$$\begin{array}{r} 47 \\ + 9 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 47. \\ + 9. \\ \hline \end{array}$$

**Multiplication:** Multiplication problems are presented as warriors at a meeting to exchange battle strategies (Manolo, 1991; Manolo et al., 2000). The warriors in the bottom set are the "experts," and are found next to the X or "times" sign. Each of these "X-perts" must meet each of the members of the group of warriors on the top row to teach them their "special battle techniques." For example, in the problem below, the "x-pert" 4 must meet with the 7, 5, 6, 8, and 3 of the top group. Students should be reminded to place ranked above ranked and unranked above unranked when they first copy down the problem. Multiplying the respective numbers and then writing the answers below the line produces the products of their planning meetings. The skills of the warriors become more specialized the farther left they are. Placing zeros whenever the student moves on to a more specialized warrior marks these specialized results.

$$\begin{array}{r} 386.57 \\ \times 23.4 \\ \hline 154628 \\ 1159710 \\ 7721400 \\ \hline 9035738 \end{array}$$

Students should be shown how to count all the unranked warriors and then count that many places from the right to the left of the final answer to place the decimal point in the answer correctly. See the example below.

$$\begin{array}{r}
 386.57 \text{ (In this problem, the student would count 3 unranked warriors)} \\
 \times 23.4 \\
 \hline
 154628 \\
 1159710 \\
 7721400 \\
 \hline
 9035.738 \text{ (Thus, the 3 unranked warriors mean that the student should} \\
 \text{place the decimal between the 5 and 7 in the answer.)}
 \end{array}$$

**Division:** In division, the students are told to imagine the warriors standing in front of a rack of various pieces of armor (Manolo, 1991; Manolo et al., 2000). Each warrior is trying on a set of armor inside the cupboard, and the cupboard is represented by the long division symbol itself or  $\sqrt{\quad}$ . However, only ranked warriors may try on armor, so if the divisor has a dot (or a decimal), the dot must be moved all the way to the right. Also, the dot in the cupboard must be shifted the same number of places to the right. Thus, division problems are changed as indicated below.

$.08\sqrt{34.26}$  becomes  $08\sqrt{3426}$  with all the decimals shifted two spots to the right.

Additional pieces of armor can be added to the rack if there is not enough armor already in the cupboard to shift the decimal the number of spaces needed. Students should be told that a decimal above the cupboard separates "expensive" armor from "inexpensive" armor. The warrior tries on the first piece of armor (in this problem, the size of that piece of armor is 3) and finds that it is too small to fit a big warrior (who in this problem is size 8). Therefore the warrior gets the next piece also, and the size now becomes 34. To tell how well the armor fits, the students should use a multiplication chart to look under the warrior's size (8) and go down that row in the table to find the size of armor closest to size 34 but not more than 34. Students should write the "measure of fit" below the armor piece in the question, then go across the table to find the number of times the warrior fitted the armor (in this example, it is 4).

Students should be told to write that number above the armor size in the cupboard, above the second digit in 34. At that point the student will have written:  $08\sqrt{3486}$

To find out how much of the armor was not used, students should subtract the "measure of fit" from the pieces of armor being tried on. This leftover number (in this case, 2) should be checked to make sure it is not larger than the size of the warrior. The next piece of armor is then brought down from the cupboard and the warrior tries it on, looking for the "measure of fit" once again. This process is repeated until the warrior has tried on each piece of armor.

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Source: Adapted from Manolo, 1991; Manolo et al., 2000.

Various research over the last two decades has shown that process mnemonics such as this is very effective for students with difficulties in math (Higbee, 1987; Manolo, 1991; Manolo et al., 2000). This tactic increases students' motivation in that the use of "warriors" makes math operations seem similar to many of the movies and video games that are so popular today. The metaphors involving warriors and their battles facilitate better retrieval of correct procedures.

Another reason why process mnemonics is so effective is that this tactic actively engages numerous fundamental mental principles of learning and memory, including organization of the constructs, association with well-understood concepts (e.g., warriors have enemies and battles), attention, and visualization. Higbee (1987) also suggests that process mnemonics provides structure that makes better sense to students. It uses concrete associations to like abstract symbols into a cohesive combination of relevance. For these reasons, teachers moving into differentiated math classes should consider using this innovative tactic for either tear-out or mainline instructional groups.

### **Visualization**

As noted previously, there does seem to be a relationship between visual processing skills in the brain and number sense and early mathematics readiness skills. For that reason, various researchers have explored the use of visualization in mathematics instruction (Foegen, 2008; Sousa, 2008). Visualization has been used to problem solve from the middle-elementary grades and up (Foegen, 2008), and the visualization tactic allows a teacher to cognitively guide the student during the math problem, and thus help the student to bridge the gap between concrete or representational understandings, and more abstract problem solving. In fact, teaching children how to visualize the problem graphically when problem solving can help them to make sense of the problem and develop increased abstract thinking ability. Thus, the intentional construction of an "image" of the problem will typically enhance a student's conceptual understanding of the problem.

Several researchers have suggested a cognitive guided inquiry method to assist students in visualizing problems (Behrend, 2003; Foegen, 2008). All children, including children who may struggle in math, have some natural problem-solving abilities, and Behrend (2003) suggested that teachers begin with the problem-solving strategies that students generate naturally. From that basis, teachers will use a series of focused questions to help students visualize the problem more completely. In this technique, the teacher asks focused questions that draw the students' attention to

various aspects of the math problem. As an example, a series of such focused questions is presented in the teacher-student dialogue in Box 5.5.

### BOX 5.5 Cognitive Guided Visualization Example

**Problem:**

Maria has 4 bags of cookies. There are 3 cookies in each bag. How many cookies in all?

**Dialogue:**

Teacher: Jay, tell me about how you got your answer of 7.

Jay: I didn't really know how to do it so I said 4 and 3 makes 7, so 7 is my answer.

Teacher: Does that tell you how many cookies there are? Would you like me to read the problem again?

Jay: OK.

Teacher: Think about it. There are 4 bags of cookies. Got that in your head?

Jay: Yeah. (He might grab for counters.)

Teacher: And there are 3 cookies in each bag.

Jay: Three in each bag?

Teacher: Yes, 3 cookies in each bag.

Jay: OK, so I need some more of these [counters].

(Jay puts 3 "cookies" in each of 4 "bags" represented by circles on a piece of paper.)

Teacher: So, how many cookies are there?

(Jay counts the "cookies" and finds that...)

Jay: There are 12 cookies!

Teacher: That's right! There are 12 cookies in all.

As shown in this dialogue, the visualization begins with students' natural attempts to problem solve. Next, teachers should ask focused questions to assist students in understanding the problem more thoroughly in order to help them visualize it. Over time, being reminded of the conditions in a particular type of problem coupled with visualization of the problem

will strengthen a student's problem-solving ability. The use of visualization also helps students generalize these newly awakened problem-solving strategies. Further, focusing on problem visualization even allows students to solve multiple-step problems that include extraneous information, because the visualization assists students to attend to only relevant information (Behrend, 2003; Carpenter, Fennema, & Franke, 1996).

Thus, instead of automatically assuming that their own answer is wrong—as many struggling students do—students can become more confident in justifying how they arrived at their own answer. When students explain their own answers to others, they and others can often spot errors that have been made and correct them. The group work basis of this tactic makes it very effective in a differentiated math class, as either a tear-out tactic or as a tactic for use with the mainline instructional group.

The implementation of cognitively guided visualization is simple, but it will be different for every student or group of students. First, working in a small tear-out group, the teacher would provide a problem for the students to consider. Next, the teacher should allow students to solve the problem in their own way. Then, the teacher will have the students share the strategies they used. Finally, using focused questions with students who got an incorrect answer, the teacher should guide them through the visualization process to arrive at the correct answer. In fact, after students have participated in this method, it should be possible for the teacher to use students for the final step above—having students ask the “guiding” questions. Initially, this should be done under the supervision of the teacher, but as students become fluent in this technique, it is possible to use this in a tear-out group, with an experienced student doing the guiding.

Behrend (2003) used this strategy to assess whether visualization would foster more accurate problem solving in two elementary-age students who were having difficulty in math. The two students explored a problem similar to the one in the dialogue above, and each devised a strategy to answer the problem. Each answer was incorrect. By encouraging one student's desire to explore mathematics and teaching the other student to “think more and guess less,” Behrend demonstrated that those students with math deficits could be taught to use visualization to make sense of the word problems. They learned how to model the problem situation using manipulatives to aid their own visualization, and were thus more actively engaged in the problem than they would have been had the problem been presented merely as a paper-and-pencil task. Further, when errors were made, they were able to realize where and why their own problem solving broke down and to see how to solve future problems correctly.

As this example illustrates, cognitive guided visualization is a technique that can assist students in moving from concrete or representational

mathematics toward deeper, more abstract thinking (Foegen, 2008). Also, research has shown that these techniques result in greater conceptual understanding of math than merely rote memory or static, “practice sheet” types of instruction. Clearly, this is the direction emphasized by the Common Core, as discussed in Chapter 1.

### Teaching Cue Words

In helping students understand word problems, many teachers begin by teaching certain cue words that represent operations. Words such as “more than,” “less than,” “extra,” “gave away,” and “add” often represent specific operations, and learning these can be a first step in problem solving. In fact, in almost all word problems, certain terms are used that are suggestive of various operations, and word problems generally involve translation of these cue words into mathematical operations. Many teachers find that students are more motivated when they are taught that solving word problems really involves a “secret code” or set of cue words that the teacher can teach them. Because these words do not always have the same meaning in all word problems, teachers should caution students to verify their interpretation of cue words by other information within the problem. At the very least, the teacher can assure students that the cue words “often indicate” a particular operation.

Initially, during instruction involving cue words, the teacher should select only problems in which the cue words mean the obvious operations. The student in Grades 3 through 5 should be trained to use the cue words and write a “number sentence” to represent a single operation problem. Then multiple operation problems that require multiple number sentences are introduced. When grading this work, teachers may wish to assign a certain number of points to correctly identifying this number sentence independent of points awarded for problem solution.

Teachers using the cue word strategy may wish to prepare a poster of these cue words and place it in front of the class, and refer to it often when discussing word problems. When a word problem is initially read in class, the teacher should challenge the students to find the cue words in the problem. Each of those should then be investigated to determine its meaning in that particular problem.

*Cognitive guided visualization is a technique that can assist students in moving from concrete or representational mathematics toward deeper, more abstract thinking.*

*Many teachers find that students are more motivated when they are taught that solving word problems really involves a “secret code” or set of cue words that the teacher can teach them.*

Finally, teachers should require that students complete some problems in which the cue words represent different operations from those they typically represent, and offer instruction on how the meaning was changed based on sentence construction in the problem. Initially, however, the teacher should present a simple word problem. Here is an example:

*Both Alonzo and his sister, Mia, had an opportunity from the teacher at school to take home some extra crackers. Now, Alonzo knew that his mom loved him and worked hard to feed the family each day, so he decided he'd bring her 10 crackers to show his love in return. From his teacher, he took 14 crackers, and Mia picked up 6 more. When Alonzo told Mia about his plans to give some crackers to Mom, he and Mia agreed to add their crackers together, take 10 crackers home for their mom, and split the rest. With that in mind, how many crackers could each one eat before getting home?*

Next, the teacher should ask the students what words in the problem might be cues on how to solve the problem. The students and teacher should note the terms (i.e., more, how many, add). The teacher may wish to discuss that the term “more” may often mean addition, as it does here, whereas phrases like “how many crackers could each one eat” might suggest subtraction.

Differentiation using the cue word strategy is not difficult. After the introduction of the sample problem described above and before the teacher-led instruction, the teacher should use the Guess, Assess, and Tear Out tactic described in Chapter 2 to identify a group of students for a differentiated activity. While the teacher conducts the lesson for the remaining students in the mainline group, the tear-out group could do a related activity. These activities may include the following:

One subgroup “analyzes” a different problem by noting the cue words in the problem. Depending on the level of the group overall, the teacher may provide a tiered version of this activity by either providing the list of cue words to the students in this group or not doing so. Later in the lesson, that problem could be used as a class illustration.

Another tear-out group may be instructed to make up two new word problems, using a minimum of two of the cue words in each problem from the list of cue words presented above. Next, that group should solve the problems before presenting them to the class.

### **A Word Problem Map**

Another example of a scaffold that can assist students in understanding word problems is the word problem map. As noted throughout this text, many students with learning difficulties in math have problems organizing

their thoughts during any learning task; this can be devastating to students' efforts to comprehend math word problems. These students may not understand that in most word problems there is an underlying structure that can be identified, and that identification of this structure can assist in problem solution. A number of researchers have encouraged specific instruction in identification of the overall structure of the math problem for students who are struggling in math (Gagnon & Maccini, 2001), and this has become known as the word problem map. A sample word problem map is presented in Box 5.6.

**BOX 5.6 A Word Problem Map**

**A Word Problem Map**

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Problem Number: \_\_\_\_\_

What type of problem is this?

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The problem asked for what information?

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What cue words were used?

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These cue words suggest what operations?

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Is there a particular order in which I must perform these operations?

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(At this point, the student should attempt the problem.)

Did I get an answer that seems correct?

---

---

Have I rechecked the problem to make certain I understand it, and is there anything I missed?

---

---

Source: Bender (2012a).

As indicated within this word problem map, the concept of cue words is stressed once again (Gagnon & Maccini, 2001). However, this tactic goes beyond merely using cue words and actually helps students structure their cognitive understanding of the word problem. The word problem map is intended to assist the student with a learning problem in math to organize his or her thoughts concerning the structure of the word problem. Thus, when a student reads a math problem in the general education classroom, he or she should be required to simultaneously complete the word problem map as a scaffold on which to build understanding of that problem.

This word problem map activity is also very effective when completed as a buddy activity; two or three students may partner together to complete the map. Furthermore, these word problem maps should be reviewed in class as a post-problem activity to check for accuracy and for comprehension of the problem.

The word problem map may be adapted as necessary across the grade levels, and teachers should feel free to implement this in any fashion they desire that works for their students. Finally, the word problem map can be used as a study guide for any future tests on that content. In fact, a wide variety of instructional activities can be built around the word problem map concept.

### **DIFFERENTIATED INSTRUCTIONAL PLAN: GRADES 3 THROUGH 6**

As mentioned in the previous chapter, I intend to make some general recommendations as to how teachers may wish to use these strategies to differentiate math classes at the various grade levels. Once again, every class is different and teachers have varying comfort levels with the different instructional ideas, so these suggestions should be used merely as guidelines. In some of the cases below, I recommend that teachers merely continue with the recommendations made for earlier grades. A differentiated instruction plan for math classes in Grades 3 through 6 is presented in Teaching Tip 5.4.

#### **Teaching Tip 5.4 Differentiated Instructional Plan: Grades 3 Through 6**

These guidelines are based on the assumption that teachers have explored many of the differentiated instructional options presented in Box 4.5 in the previous chapter.

##### **Continue Khan Academy**

I recommended that teachers throughout the grade levels set up every member of their class within Khan Academy, and allocate 15 minutes daily work for this.

### **Continue Instructional Gaming in Math**

Teachers should find a gaming website that meets their needs, and have students practice mathematics using educational games, using those several times each week. This will facilitate positive experiences in mathematics.

### **Flip the Math Class**

Beginning as early as Grade 3, but certainly by Grade 4, teachers should experiment with flipping the mathematics class, as described in Chapter 3. I recommend that teachers begin a new content or conceptual area by having students view a video demonstration as homework, and use the next math class period as a peer-buddy practice time on that type of problem. I'd recommend such peer-buddy activity for 20 minutes daily.

Ideally, all initial instruction in mathematics could be completed in a "flipped" fashion by Grade 5, saving teachers a great deal of instructional time in the math class.

### **Set Up a Blog**

Beginning in Grade 3, if not before, teachers should have students working within a class blog. This will facilitate much student-to-student coaching on mathematics problems, and should foster increased engagement. Teachers may require students to make a blog posting a minimum of twice a week, as an out-of-class assignment.

### **Teach With a Mathematics Wiki**

At least by Grade 4, teachers should migrate from the class blog to a wiki format to increase students' social interaction and communication on mathematics. Students should interact with each other via the class wiki on a daily basis for 10 to 15 minutes.

### **Continue Using Student Avatars**

Many students enjoy using avatars, and if students are used to this, I suggest that avatars be used throughout the elementary math years.

### **Devise a Range of Tier 2 and Tier 3 Intervention Options**

Any of the scaffolded interventions described in this chapter could become excellent interventions, since scaffolded instruction often means individual instruction anyway. Using word problem maps, process mnemonics, or graphic illustrations can greatly assist students struggling in math, and thus these make effective interventions at the Tier 2 and 3 levels.

By using this differentiated instructional plan, all students will receive exactly the instruction they need, at their level, and the fears and anxieties associated with mathematics will be alleviated, based on the fun associated with math games and successful experiences.

## WHAT'S NEXT?

This chapter has presented a variety of differentiation strategies for the mid-elementary grades, including more advanced strategies for increasing number sense in 8- to 11-year-old students. Within the context of constructivist theory, a variety of additional tech-based teaching strategies were discussed including blogs and wikis for the math class. Also, several scaffolded instruction tactics were presented, including graphic representations for math facts, process mnemonics, cue words, and word problem maps. As in the previous chapter, a differentiated instructional plan was presented with specific recommendations about structuring the differentiated mathematics class in the primary grade levels.

In the next chapter, the focus is on the higher elementary and middle school grades. Social networking options will be described, as many teachers in middle school are using social networking as a teaching tool, and cognitive instructional strategies for the early teen years will be presented, that will assist in developing conceptual understanding in mathematics.