

PRIMARY MATHEMATICS



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PRIMARY MATHEMATICS

Teaching Theory and Practice



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About this book

This book has been written to satisfy the needs of all primary trainees on all courses of initial teacher training in England and other parts of the UK where a secure knowledge and understanding of how to teach mathematics is required for the award of Qualified Teacher Status (QTS) or its equivalent. This book will also be found useful by Newly Qualified Teachers (NQTs), mentors, curriculum co-ordinators and other professionals working in education who have identified aspects of their mathematics practice which require attention or who need a single resource to recommend to colleagues.

Features of the chapters of this book include:

- clear links with the *Teachers' Standards* in England;
- references to required mathematics knowledge and understanding;
- research summaries that give insights into how the theory of mathematics teaching has developed, including seminal studies on children's ideas about mathematics and how their understanding develops;
- examples of practice in the classroom to illustrate important points;
- suggestions for embedding ICT in your practice;
- reminders of how planning for teaching mathematics fits in with the bigger picture, across the curriculum and with other aspects of school life;
- reflective tasks and practical activities;
- a summary of key learning points for each chapter;
- references to key texts and suggestions for further reading.

The first section, 'Essentials of teaching theory and practice for primary mathematics', which explores teaching strategies, planning, assessment and some common misconceptions, gives you focused practical advice for teaching. Also included here is a chapter on 'Mathematics in the Early Years Foundation Stage' to help you to develop an understanding of good practice in teaching children as they start school. The second section, 'Progression in children's understanding of mathematics', includes details of how children's understanding progresses in some key areas of the curriculum.

For those undertaking credits for a Master's Degree, we have included suggestions for further work and extended study at the end of some chapters in a section called 'M-Level Extension'. The book also contains a Glossary of terms.

What is primary mathematics and why is it taught?

Children need to develop a good mathematical understanding in order to function effectively as members of our society. The expression ‘functional literacy’ is frequently used when discussing children learning aspects of English language and literacy. However, we also need to consider ‘functional numeracy’, that which is required by children in order to operate and interact effectively in today’s society. The number of occasions throughout the day when we all encounter mathematical concepts is manifold. An understanding of different aspects of number is required to find the correct house in the street, to call someone on the phone or to lay the table. Measures are used every day, either accurate or estimated, when shopping (mass, length, capacity, money), when driving a car, riding a bike or crossing the road (speed, distance, time) and when telling the time or acknowledging the passing of time. We live in a three-dimensional world, hence an understanding of spatial concepts is vital if we are to interact with and make sense of our physical environment. Each day we encounter data, we evaluate data and we interpret data. All of this demonstrates just how important for children a clear knowledge and understanding of mathematics really are.

In order to achieve the aim of ‘functional numeracy’ children need to be able to think flexibly and to apply their knowledge to new situations, to solve practical problems, to experiment within mathematics itself, to develop the ability to reason mathematically and to communicate their reasoning to others. We cannot assume a child is ‘functionally numerate’ if they can only answer pages of questions. They need to be able to abstract and generalise from specific situations in order to demonstrate their mathematical thinking. These elements of generalising and communicating mathematical thinking need to be foremost in our teaching. However, we must also not lose sight of the awe and wonder of mathematics, the creativity and elegance that ensure the stimulation, challenge and enjoyment.

RESEARCH SUMMARY

In order to achieve the aim of ‘functional numeracy’ successfully we need to consider how we are building a community of learners within our mathematics classrooms. Bruce (2007) identified student interaction as one of the essential

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characteristics of effective mathematics teaching. However, she also found that, left to their own devices, the children did not necessarily engage in high-quality maths-talk. She identifies five features that encourage high-quality student interactions, that establish a learning community within the mathematics classroom and that increase levels of achievement in mathematics.

1 **The use of rich maths tasks**

When a task allows for multiple strategies and/or has multiple solutions, children have greater opportunity to explain and justify their reasoning.

2 **Justification of solutions**

Encouraging the children to engage in productive arguments and justification in class discussion, rather than simply recounting procedures, leads to greater understanding.

3 **Students questioning one another**

Another powerful strategy is getting the children to ask higher-order questions of one another. Using prompt cards with question stems such as '*How are ... and ... similar?*' can usefully support children in this. A child might use the prompt here to ask '*How are squares and parallelograms similar?*' within the context of geometry. The children retained more when using prompt cards than when they spent the same amount of time in small group discussion without prompts.

4 **Use of wait time**

Asking questions that require higher-level thinking is only useful if children are given sufficient time to do the related thinking. Children's attitude towards learning improves if this wait time is combined with higher-level questioning.

5 **Use of guidelines for maths-talk**

Explain: 'This is what we ...'

Agree with reason: 'I agree because ...'

Disagree with reason: 'I disagree because ...'

Build on: 'I want to build on ...'

Go beyond: 'That makes me think about ...'

Wait time

The use of appropriate guidelines can help teachers and children engage in high-quality interactions leading to richer mathematical thinking and deeper understanding.

Throughout this book we aim to support you in developing successful approaches to teaching mathematics, in building communities of learners within your classrooms and ensuring appropriate levels of attainment in mathematics for the children you teach.

The *Teachers' Standards*

This book is written to support the development of the knowledge and skills required by teachers in order to demonstrate attainment in the *Teachers' Standards* (DfE, 2011b).

In order to demonstrate this attainment, it is important to know and understand certain things by the time a course of initial teacher training is completed. Within mathematics these include:

- the key aspects of mathematics underpinning children's progress;
- methods of developing children's mathematical knowledge, understanding and skills;
- how to plan and pace mathematics lessons;
- the selection and use of mathematical resources;
- how to lead oral work and use interactive methods;
- recognising common mathematical errors and misconceptions and how to prevent and remedy them;
- assessing and evaluating mathematics teaching and learning;
- recognising standards of attainment in mathematics that should be expected of pupils;
- the importance of engaging pupils' interest in and enthusiasm for mathematics.

If you feel that you need more help with your own knowledge and understanding, see the companion title in this series, Primary Mathematics: Knowledge and Understanding (8th edn) (Learning Matters, 2018).

Curriculum context

This book will support you to teach the mathematics early learning goals in the Early Years Foundation Stage and the Key Stages 1 and 2 content of the mathematics National Curriculum for England (DfE, 2013a). Because schools have the freedom to design their own school curriculum, which might include aspects of mathematics beyond the National Curriculum, we have focused in this book on giving you the insights into the development of the theory behind the core areas of mathematics teaching that you will need to inform your practice, as you plan to promote the mathematical understanding of the children you work with and to ensure that they develop effective skills of using and applying mathematics.

Early Years Foundation Stage

The *Statutory Framework for the Early Years Foundation Stage* (EYFS) (DfE, 2014) sets the standards that all early years providers must meet to ensure that children learn and develop well and are kept healthy and safe. It promotes teaching and learning to ensure children's 'school readiness' and gives children the broad range of knowledge and skills that provide the right foundation for good future progress through school and life. Children will remain in the EYFS until the end of the academic year in which they turn five years of age. In practice this will mean that children will follow EYFS until they finish their Reception year.

There are seven areas of learning and development that shape the educational programmes in early years settings. The three *prime* areas are:

- communication and language;
- physical development; and
- personal, social and emotional development.

The four *specific* areas are:

- literacy;
- mathematics;

- understanding the world; and
- expressive arts and design.

For the purposes of this book we shall focus on the area of learning and development ‘mathematics’, which *involves providing children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems; and to describe shapes, spaces, and measures* (DfE, 2014, p. 8). Crucially, the focus in mathematics is that children talk about and apply mathematics in a way that makes common sense to them (Tickell, 2011, p. 103).

Mathematics in the National Curriculum

The mathematics primary National Curriculum in England is split into three key stages: Key Stage 1 (5–7 years), lower Key Stage 2 (7–9 years) and upper Key Stage 2 (9–11 years). Each key stage includes attainment targets and by the end of each key stage, *pupils are expected to know, apply and understand the matters, skills and processes specified in the mathematics programme of study* (DfE, 2013a, p. 89). The mathematics National Curriculum in England has three aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Furthermore, the National Curriculum explains how mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. In order to support pupils with this, you need a good subject knowledge to see the connections, be fluent, reason mathematically and solve problems yourself.

At the time of writing, the DfE have recently confirmed that the external testing for mathematics will continue at the end of Key Stage 2. In the Government’s response (DfE, 2011b) to the Bew Review (2011) into Key Stage 2 testing, assessment and accountability they stated:

Introduction

It is important that mathematics tests are accessible to all pupils and do not unfairly disadvantage weaker readers. At the time of publication, The Standards and Testing Agency was reviewing all future National Curriculum Tests in mathematics to ensure that they remain accessible to all pupils, and that they are primarily tests of mathematics rather than reading.

(DfE, 2011b, p. 10)

In September 2017, the Department for Education published the *Primary Assessment in England: Government Consultation Response*. This document outlines the government response to the consultation on the long-term future of primary assessment that ran from March 2017 to June 2017. This consultation:

sought views on a number of key proposals about the future of the statutory primary assessment system. These included assessment in the early years, the starting point for measuring the progress that pupils make at primary school, statutory end-of-key stage teacher assessment and proposals to ensure that we have a proportionate assessment system.

(DfE, 2017)

The document gives details of the government's aim to:

remove the statutory requirement for schools to report teacher assessment judgements in English reading and mathematics at the end of key stage 2 from the 2018 to 2019 academic year onwards, once the relevant legislation has been amended. We believe that removing this duty to report judgements against the statutory teacher assessment frameworks will reduce burdens for teachers.

(DfE, 2017)

It makes clear, however, that schools' statutory requirement to report pupils' attainment and progress to parents will remain.

Also outlined is the government's plan to

make assessments at the end of key stage 1 (both national curriculum tests and statutory teacher assessment) non-statutory as soon as the reception baseline assessment has become fully established. If possible, we intend to make this change from the 2022 to 2023 academic year onwards.

(DfE, 2017)

Finally, the document explains clear plans to introduce the Multiplication Tables Check (a statutory test announced in 2016) at the end of Year 4.

REFERENCES

Bew, P. (2011) *Independent Review of Key Stage 2 Testing, Assessment and Accountability: Final Report*. London: DfE.

Bruce, C. (2007) 'Student Interaction in the Math Classroom: Stealing Ideas or Building Understanding'. *What Works? Research into Practice*. Toronto: Literacy and Numeracy Secretariat.

DfE (2011a) *Independent Review of Key Stage 2 Testing, Assessment and Accountability: Government Response*. London: DfE.

DfE (2011b) *Teachers' Standards*. Available at www.gov.uk/government/publications/teachers-standards (accessed 13/4/14).

DfE (2013a) *The National Curriculum in England: Framework document*. London: DfE. Available at: www.gov.uk/government/uploads/system/uploads/attachment_data/file/210969/NC_framework_document_-_FINAL.pdf (accessed 13/4/14).

DfE (2013b) *Primary Assessment and Accountability under the New National Curriculum*. London: DfE. Available at: www.gov.uk/government/uploads/system/uploads/attachment_data/file/298568/Primary_assessment_and_accountability_under_the_new_curriculum_consultation_document.pdf (accessed 13/4/14).

DfE (2014) *Statutory Framework for the Early Years Foundation Stage: Setting the standards for learning, development and care for children from birth to five*. London: DfE. Available at: http://www.gov.uk/government/uploads/system/uploads/attachment_data/file/299391/DFE-00337-2014.pdf (accessed 23/4/14).

DfE (2017) *Primary assessment in England: Government consultation response*. London: DfE. Available at: www.gov.uk/government/uploads/system/uploads/attachment_data/file/644871/Primary_assessment_consultation_response.pdf

DfES (2003) *Excellence and Enjoyment – A Strategy for Primary Schools*. London: DfES.

Tickell, C. (2011) *The Early Years: Foundations for Life, Health and Learning. An Independent Report on the Early Years Foundation Stage to Her Majesty's Government*. London: DfE.

FURTHER READING

To support you in understanding the curriculum context, you may find it helpful to refer to some of the following documentation:

DfE (2013) *Mathematics Programmes of Study: Key Stages 1 and 2. National Curriculum for England*. London: DfE. Available at: www.gov.uk/government/uploads/system/uploads/attachment_data/file/239129/PRIMARY_National_Curriculum_-_Mathematics.pdf

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DfE (2014) *Statutory Framework for the Early Years Foundation Stage: Setting the standards for learning, development and care for children from birth to five*. London: DfE. Available at: www.gov.uk/government/uploads/system/uploads/attachment_data/file/299391/DFE-00337-2014.pdf (accessed 23/4/14).

Ofsted (2009) *Mathematics: Understanding the Score. Improving practice in mathematics teaching at primary level*. London: Ofsted.

Williams, P. (2008) *Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools*. Nottingham: DCSF.

In addition, you may want to use the websites of the following mathematics education organisations, which have resource materials, online discussions and information on upcoming courses and conferences:

The Association of Teachers of Mathematics: www.atm.org.uk

The Mathematical Association: www.m-a.org.uk

The National Centre for Excellence in the Teaching of Mathematics: www.ncetm.org.uk