CHAPTER 1

Introducing the Five Practices

Many teachers are daunted by an approach to pedagogy that builds on student thinking. Some are worried about content coverage, asking, "How can I be assured that students will learn what I am responsible for teaching if I don't march through the material and tell them everything they need to know?" Others—teachers who perhaps are already convinced of the importance of student thinking—may be nonetheless worried about their ability to diagnose students' thinking on the fly and to quickly devise responses that will guide students to the correct mathematical understanding.

Teachers are correct when they acknowledge that this type of teaching is demanding. It requires knowledge of the relevant mathematical content, of student thinking about that content, and of the subtle pedagogical "moves" that a teacher can make to lead discussions in fruitful directions, along with the ability to rapidly apply all of this in specific circumstances. Yet, we have seen many teachers learn to teach in this way, with the help of the five practices.

We think of the five practices as skillful improvisation. The practices that we have identified are meant to make student-centered instruction more manageable by moderating the degree of improvisation required by the teacher during a discussion. Instead of focusing on in-the-moment responses to student contributions, the practices emphasize the importance of planning. Through planning, teachers can anticipate likely student contributions, prepare responses that they might make to them, and make decisions about how to structure students' presentations to further their mathematical agenda for the lesson. We turn now to an explication of the five practices.

The Five Practices

The five practices were designed to help teachers to use students' responses to advance the mathematical understanding of the class as a whole by providing teachers with some control over what is likely to happen in the discussion as well as more time to make instructional decisions by shifting some of the decision making to the planning phase of the lesson. The five practices are—

1. ***anticipating*** likely student responses to challenging mathematical tasks and questions to ask to students who produce them;

2. ***monitoring*** students' actual responses to the tasks (while students work on the tasks in pairs or small groups);
3. ***selecting*** particular students to present their mathematical work during the whole-class discussion;
4. ***sequencing*** the student responses that will be displayed in a specific order; and
5. ***connecting*** different students' responses and connecting the responses to key mathematical ideas.

Each practice is described in more detail in the following sections, which illustrate them by identifying what Mr. Crane could have done in the Leaves and Caterpillars lesson (presented in the introduction), to move student thinking more skillfully toward the goal of recognizing that the relationship between caterpillars and leaves is multiplicative, not additive.

Anticipating

The first practice is to make an effort to actively envision how students might mathematically approach the instructional task or tasks that they will work on and consider questions that could be asked of the students who used specific strategies. This involves much more than simply evaluating whether a task is at the right level of difficulty or of sufficient interest to students, and it goes beyond considering whether or not they are getting the "right" answer.

Anticipating students' responses involves developing considered expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn.

Anticipating requires that teachers do the problem as many ways as they can. Sometimes teachers find that it is helpful to expand on what they might be able to think of individually by working on the task with colleagues, reviewing responses to the task that might be available (e.g., work produced by students in the previous year, responses that are published along with tasks in supplementary materials), and consulting research on student learning of the mathematical ideas embedded in the task. For example, research suggests that students often use additive strategies (such as Missy and Kate's response, shown in fig. 0.2) to solve tasks like the Leaves and Caterpillars problem, in which there is a multiplicative relationship between quantities (Hart 1981; Heller et al. 1989; Kaput and West 1994).

Anticipating solution strategies in advance of the lesson would have made it possible for Mr. Crane to carefully consider what actions he might take should students produce specific solutions. For example, anticipating that some students would use the additive strategy would have made it possible for Mr. Crane to recognize it when his students produced it and carefully consider the questions he could ask to help make the students aware of the multiplicative nature of the relationship between the caterpillars and leaves.

In addition, if Mr. Crane had solved the problem ahead of time in as many ways as possible, he might have realized that there were at least three different correct strategies for arriving at the correct answer—scaling up, scale factor, and unit rate—and that each of these could be expressed with different representations (pictures, tables, and written explanations).

Monitoring

Monitoring student responses involves paying close attention to students' mathematical thinking and solution strategies as they work on the task. Teachers generally do this by circulating around the classroom while students work either individually or in small groups. Carefully attending to what students do as they work makes it possible for teachers to use their observations to decide what and whom to focus on during the discussion that follows (Lampert 2001).

One way to facilitate the monitoring process is for the teacher to use a monitoring chart as shown in figure 1.1. Before beginning the lesson, the teacher can list the solutions that he or she anticipates that students will produce that will help in accomplishing the mathematical goals for the lesson. A list of solutions, such as the one shown in column 1 of the chart in figure 1.1 for the Leaves and Caterpillars task, can help the teacher keep track of which students or groups produced which solutions or brought out which ideas that he or she wants to make sure to capture during the whole-group discussion. The "Other" cell in the first column provides the teacher with the opportunity to capture ideas that he or she had not anticipated.

As discussed in the introduction, Mr. Crane's lesson provided limited, if any, evidence of active monitoring. Although Mr. Crane knew who got correct answers and who did not and that a range of strategies had been used, his choice of students to present at the end of the class suggests that he had not monitored the specific mathematical learning potential available in any of the responses. What Mr. Crane could have recorded while students worked on the task is shown in the third column in the chart in figure 1.1.

It is important to note, however, that monitoring involves more than just watching and listening to students. During this time, the teacher should also ask questions that will make students' thinking visible, help students clarify their thinking, ensure that members of the group are all engaged in the activity, and press students to consider aspects of the task to which they need to attend. Many of these questions can be planned in advance of the lesson, on the basis of the anticipated solutions, as shown in the second column of figure 1.1. For example, if Mr. Crane had anticipated that a student would use a unit-rate approach (Janine's or Kyra's responses—see fig. 0.3), reasoning from the fact that the number of leaves eaten by one caterpillar was 2½, then he might have been prepared to question, say, for example, Janine, regarding how she came up with the number 2½ and how she knew to multiply it by 12. In addition, Mr. Crane might have asked Janine questions that would prompt her to reflect on the relationship between the two ratios (e.g., "First you had 2C and 5L. Now you have 12C and 30L. How are these related?") or apply her method to a new situation (e.g., "Suppose you had to feed 100 caterpillars instead of 12. How many leaves would they need?"). Questioning a student or group of students while they are exploring the task provides them with the opportunity to refine or revise their strategy prior to whole-group discussion and consider mathematical relationships and also gives the teacher insight regarding what the student understands about the problem and the mathematical ideas embedded in it. The monitoring chart provides a record of "who did what" which can then be used to make decisions regarding the solutions and ideas to highlight during the whole class discussion.

Strategy	Questions	Who and What	Order
Unit rate Find the number of leaves eaten by one caterpillar (2½) and multiply by 12, or add the amount for one 12 times to get 30 leaves.	• How did you get 2½? What does it represent? • Why did you multiply by 12? What does it represent? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had to feed 100 caterpillars instead of 12. How many leaves would they need?*	Janine: multiplied 12 x 2½ (sticks representing caterpillars) Kyra: added 2½ 12 times (picture of leaves and caterpillars)	Janine – 3rd
Scale Factor Find that the number of caterpillars (12) is 6 times the original amount (2), so the number of leaves (30) must be 6 times the original amount (5).	• What does 6 represent? • Why do you multiply by 5? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had 100 caterpillars instead of 12. How many leaves would they need?*	Jason: narrative description (language a little confusing—*count by twos until you come to half of 12*)	Jason – 4th
Scaling Up Increase the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillars until you reach 30 leaves.	• How did you get 30 leaves? • How do you know that this is the right number of leaves? • *First you had 2C and 5L. Now you have 12C and 30L. How are these related?* • *Suppose you had 100 caterpillars instead of 12. Could you find the number of leaves needed without continuing your drawing or table?*	Jamal: table with leaves and caterpillars increasing in increments of 2 and 5 Martin and Melissa: 6 sets of leaves and caterpillars	Martin – 1st Jamal – 2nd
Additive Find that the number of caterpillars has increased by 10 (2 + 10 = 12), so the number of leaves must also increase by 10 (5 + 10 = 15).	• Why did you add 10 to 2 and 5? • How many leaves did each caterpillar get when there were only 2 caterpillars? • How many leaves does each caterpillar get now that there are 12 caterpillars? • *What if we want each caterpillar to get the same number of leaves no matter how many caterpillars we have? What could you do?*	Missy and Kate	
OTHER Multiply leaves and caterpillars together: 5 x 12 = 60.		Darnell and Marcus	

Fig. 1.1. A chart for monitoring students' work on the Leaves and Caterpillars task (gray shading indicates what could be anticipated before teaching the lesson)

Selecting

Having monitored the available student strategies in the class, the teacher can then select particular students to share their work with the rest of the class to get specific mathematics into the open for examination, thus giving the teacher more control over the discussion (Lampert 2001). The selection of particular students and their solutions is guided by the mathematical goal for the lesson and the teacher's assessment of how each contribution will contribute to that goal. Thus, the teacher selects certain students to present because of the mathematics in their responses.

A typical way to accomplish "selection" is to call on specific students (or groups of students) to present their work as the discussion proceeds. Alternatively, the teacher may let students know before the discussion that they will be presenting their work. In a hybrid variety, a teacher might ask for volunteers but then select a particular student that he or she knows is one of several who have a particularly useful idea to share with the class. By calling for volunteers but then strategically selecting from among them, the teacher signals appreciation for students' spontaneous contributions, while at the same time keeping control of the ideas that are publicly presented.

Returning to the Leaves and Caterpillars vignette, if we look at the strategies that were shared, we note that Kyra and Janine had similar strategies that used the idea of unit rate (i.e., finding out the number of leaves needed for one caterpillar). Given that, there may not have been any added mathematical value to sharing both. In fact, if Mr. Crane wanted students to see the multiplicative nature of the relationship, he might have selected Janine, since her approach clearly involved multiplication.

Also, there might have been some payoff from sharing the solution produced by Missy and Kate (fig. 0.2) and contrasting it with the solution produced by Melissa (fig. 0.3). Although both approaches used addition, Missy and Kate inappropriately added the same number (10) to both the leaves and the caterpillars. Melissa, on the other hand, added 5 leaves for every 2 caterpillars, illustrating that she understood that this ratio (5 for every 2) had to be kept constant.

Sequencing

Having selected particular students to present, the teacher can then make decisions regarding how to sequence the student presentations. By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances of achieving their mathematical goals for the discussion. For example, the teacher might want to have the strategy used by the majority of students presented before those that only a few students used, to validate the work that the majority of students did and make the beginning of the discussion accessible to as many students as possible. Alternatively, the teacher might want to begin with a strategy that is more concrete (using drawings or concrete materials) and move to strategies that are more abstract (using algebra). This approach—moving from concrete to abstract—serves to validate less sophisticated approaches and allows for connections among approaches. If a common misconception underlies a strategy that several students used, the teacher might want to have it addressed first so that the class can clear up that misunderstanding to be able to work on developing more successful ways of tackling the problem. Finally, the teacher might want to have related or contrasting strategies presented one right after the other in order to make it easier for the class to compare them. Again, during planning the teacher can consider possible ways of sequencing anticipated responses to highlight the mathematical ideas that are key to the lesson. Unanticipated responses can then be fitted into the sequence as the teacher makes final decisions about what is going to be presented.

More research needs to be done to compare the value of different sequencing methods, but we want to emphasize here that particular sequences can be used to advance particular goals for a lesson. Returning to the Leaves and Caterpillars vignette, we point out one sequence that could have been used: Martin (scaling up by collecting sets—picture), Jamal (scaling up—table), Janine (unit rate—picture/written explanation), and Jason (scale factor—written explanation).

This ordering begins with the least sophisticated representation (a picture) of the least sophisticated strategy (scaling up by collecting sets) and ends with the most sophisticated strategy (scale factor), a sequencing that would help with the goal of accessibility. In addition, by having the same strategy (scaling up) embodied in two different representations (a picture and a table), students would have the opportunity to develop deeper understandings of how to think about this problem in terms of scaling up. Decisions regarding which solutions to share and which students should present and in what order can be recorded in the fourth column of the monitoring chart (see fig. 1.1).

Connecting

Finally, the teacher helps students draw connections between their solutions and other students' solutions as well as to the key mathematical ideas in the lesson. The teacher can help students make judgments about the consequences of different approaches for the range of problems that can be solved, one's likely accuracy and efficiency in solving them, and the kinds of mathematical patterns that can be most easily discerned. Rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal is to have student presentations build on one another to develop powerful mathematical ideas.

Returning to Mr. Crane's class, let's suppose that the sequencing of student presentations was Martin, Jamal, Janine, and Jason, as discussed above. The teacher could ask students how Martin's picture is related to Jamal's table (Jamal's table provides a running total as groups of 2 caterpillars and 5 leaves are added) and to compare Jamal and Janine's responses and to identify where Janine's unit rate (2½ leaves per caterpillar) is found in Jamal's table (it is the factor by which the number of caterpillars must be multiplied to get the number of leaves). Students could also be asked to compare Jason's explanation with Jamal and Martin's work to see if the scale factor of 6 can be seen in each of their tabular and pictorial representations (6 is the number of sets of 2 caterpillars and 5 leaves in Martin's picture and the number of leaves-to-caterpillars ratios in Jamal's table).

It is important to note that the five practices build on another. Monitoring is less daunting if the teacher has taken the time to anticipate ways in which students might solve a task and questions to ask students about their responses. Although a teacher cannot know with 100 percent certainty how students will solve a problem prior to the lesson, many solutions can be anticipated and thus easily recognized during monitoring. A teacher who has already thought about the mathematics represented by those solutions can turn his or her attention to making mathematical sense of those solutions that are unanticipated. Selecting, sequencing, and connecting, in turn, build on effective monitoring. Effective monitoring will yield the substance for a discussion that builds on student thinking, yet moves assuredly toward the mathematical goal of the lesson.

Conclusion

The purpose of the five practices is to provide teachers with more control over student-centered pedagogy. They do so by allowing the teacher to manage the content that will be discussed and how it will be discussed. Through careful planning, the amount of improvisation required by the teacher "in the moment" is kept to a minimum. Thus, teachers are freed up to listen to and make sense of outlier strategies and to thoughtfully plan connections between different ways of solving problems. All of this leads to more coherent, yet student-focused, discussions.

While it is critical for teachers to orchestrate whole-class discussions in which students present their solutions and the teacher asks questions to ensure that the solutions are connected and the mathematics is made explicit, doing so is not enough to ensure that all students are learning what was intended. The teacher needs to hold the students who did not publicly present their work accountable for listening to and making sense of what the presenters say and do. Specific ways in which teachers can hold students accountable are discussed in detail in chapter 6.

In the next chapter, we explore an important first step in enacting the five practices: setting goals for instruction and identifying appropriate tasks. Although this work is not one of the five practices, it is the foundation on which the five practices are built. In chapters 3, 4, and 5, we then explore the five practices in depth and provide additional illustrations showing what the practices look like when enacted and how the practices can lead to more productive discussions.