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Math Review, Vocabulary, and Symbols

Terms: case, subject, sample, population, statistic, parameter, variable, constant, summation

Symbols: $X, X_1, X^2, \bar{X}, M, p, q, N, n, \approx, <, >, ||, \Sigma, \sqrt{\quad}$

Learning Objectives:

- Become familiar with common statistical terms and symbols
- Review common arithmetic rules and functions

Getting Started

Statistics is a language. As with any language, before we can communicate effectively, we must master that language's vocabulary and symbol system. This first module, then, introduces essential statistical vocabulary and symbols. Additional vocabulary and symbols will be introduced as you progress through the textbook.



There still remain
three studies suitable
for man. Arithmetic is
one of them.

—Plato

Following the vocabulary and symbols, the remainder of this module is a math refresher. A surprising proportion of students need a math review or, at the least, benefit from it. Note that the review is basic. It is intended merely to make sure that everyone using this textbook—including the math phobic and those who have been out of school for some time—remembers the fundamentals.

Nevertheless, do not be misled by this math review into thinking that this is a “remedial math” type of textbook. It is not. This textbook is a comprehensive, college-level treatment of statistical concepts and calculations. While the first few modules may seem elementary, the material quickly builds in difficulty. Still, the mathematics never becomes overwhelming. The difficulty of statistics seldom lies with the mathematics. Rather, the difficulty lies with the inferential logic. Even in its most difficult modules, this textbook requires only elementary algebra. In most units, basic arithmetic is sufficient. You are fully capable of mastering the material.

Your classroom teacher is, of course, the primary instructor for your course. However, I have written this textbook in a conversational tone, as if I were standing alongside you,

guiding you in your understanding of statistics. In that sense, I, too, am your teacher. So, as your coteacher, let me offer a few words of advice.

- First, carefully read all the assigned text material. Because statistics is logic driven, missing a piece of the argument can be deadly to your later understanding. Think of this textbook as an integral part of your class instruction.
- Second, take notes. Have a highlighter and a pen nearby when reading the textbook.
- Third, when you come up against a *Check Yourself!* box, don't just skip over it. Rather, check yourself! The boxes are strategically placed throughout the text to ensure mastery of important material before progressing to the next topic.
- Finally, take time to laugh. Despite rumors to the contrary, the study of statistics doesn't have to be boring. I have peppered the text with cartoons and the margins with quips and quotes. Let these be tension breakers, reminding you that even statistics has a lighter side. For example, did you know that one of the statistics used to test experimental hypotheses was originally created to test the quality of beer? Or that the only certain conclusion with any inferential statistic is that we're not certain? Or that from knowledge of the amount of ice cream eaten by members of a community, we can predict the number of drownings in that community quite accurately?

How can these things be? You will have to read on to find out. I hope that you find your study to be . . . *Statistics Alive!*



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Vocabulary and Symbols

To work with statistics, we need to use the notation commonly accepted by those in the field. Most statistical formulas or problems contain one or more of these terms or symbols.

Case. This is an individual unit under study. It could be a person, an animal, a car, a soft drink, a lightbulb, and so on. For example, a study of car-stopping distances might include 300 cases (cars).

Subject. When the cases are human beings, they are often referred to as subjects. For example, a study of running speed in college sophomores might include 80 subjects (80 sophomore students).

Participant. This is the American Psychological Association's (APA) preferred word for "subjects," defined above.

Sample. This is the group of participants in a study. Usually, they are a subset of a larger group. For example, we might measure the height of a sample of 100 elderly women.

Population. This is the larger group of participants about which we want to draw a conclusion. For example, although we may have sampled only 100 elderly women, we might want to draw a conclusion about the height for the whole population of elderly women of which the 100 women were a part.

Statistic. This is a summary number (e.g., an average) for a sample. Our statistical average might be 63 in.

Parameter. This is a summary number (e.g., an average) for a population. Our parametric average might be 63.5 in.

Variable. When the value of the trait being measured varies from case to case, that trait is referred to as a variable. For example, when measuring the running speed of college sophomores, running speed is a variable because each student is expected to run at a different speed.

Constant. When the value of the trait being measured is the same for all cases, that trait is referred to as a constant. In a mechanized (machine-driven) study of car-stopping distance, for example, researchers might figure how long it takes the average person to raise a foot to the brake pedal and then add that value as a constant to the measured stopping distances.

Uppercase Letters. These usually represent variables, scores that vary from case to case. An example would be X , which might stand for each subject's running score. Every subject would have a different X score.

Lowercase Letters. These usually stand for constants, values that are the same for each case. An example would be c , which, in the study of car-stopping distance, stands for the average time it takes to raise a foot to the brake pedal.

Bar Over a Letter. This represents the average of a variable. An example would be " \bar{X} " (pronounced "X-bar"), which is the average score on the variable X .

M . This letter is reserved for the mean, known in lay language as the average. Wait. Didn't I say above that \bar{X} is the symbol for the average? Yes, either symbol, \bar{X} or M , is used to represent an average. However, M is the more recent symbol and is the only symbol now accepted by American Psychological Association (APA) journals. Nevertheless, you should learn to recognize the \bar{X} symbol, as it will appear in older textbooks, in textbooks aimed at students outside the social sciences, and in journal articles published prior to the change.

p . This letter is reserved for the probability of an event occurring. An example would be the probability of rolling a 3 on a standard die. Because a die has six sides, numbered one through six, the probability of rolling any given number is $1/6$. In decimal form, the probability is .167.

q . This letter is reserved for the probability of an event *not* occurring, or in other words, $1 - p$. An example would be the probability of not rolling a 3 on a standard die. The probability is $1 - p$, which is $1 - 1/6$, which is $5/6$. In decimal form, the probability is .833.

N, n . This letter is reserved for the number of cases. An example would be the number of students in your statistics class. If there are 50 people in your class, then $N = 50$. Typically, n is the number of cases in a sample, and N is the number of cases in a population.

Subscripts. Subscripts refer to particular subjects or cases. Continuing with the variable " X ," " X_1 " would be the score of the first subject, " X_2 " the score of the second subject, and so on.

Wavy Parallel Lines (\approx). This symbol means *about* or *approximately*. For example, we might say that the coat cost $\approx \$200$ or that you expect to experience a snack attack in ≈ 15 min.

Less Than and Greater Than ($<$ and $>$). These symbols mean *less than* and *greater than*, respectively. For example, we might say that the coat cost $< \$200$ or that you expect > 15 min to pass before you experience a snack attack.

Summation (Σ). This means to add the scores of all cases. For example, ΣX means to add up all scores on the variable X .

Multiplication Indicators. There are several ways to indicate that two numbers are to be multiplied. As a child, you learned to write “ 3×4 .” However, when substituting letters for numerals in algebra, the use of “ \times ” to indicate multiplication becomes confusing. Thus, a second method to indicate multiplication is to put parentheses around each number individually. For example, “ $(3)(4)$ ” means to multiply 3 and 4. Another way is to put a midlevel dot or asterisk between the numbers. For example, “ $3 * 4$ ” means to multiply 3 and 4. A fourth way, which can be used only when the numbers are symbolized by letters, is to write the numbers next to each other without a space. For example, “ ab ” means to multiply a and b .

Reciprocal. A reciprocal is “one divided by the number.” Thus, the reciprocal of 3 is $1/3$, and the reciprocal of 6 is $1/6$.

Superscripted Number or Exponent. This indicates the number of times a number should be multiplied by itself. For example, 3^2 (pronounced “three squared”) means to multiply 3 twice, which is $3 * 3$, which is 9.

Radical Sign ($\sqrt{\quad}$). This says to find the square root of the number under the radical sign. The square root is the number that when multiplied by itself yields the number under the radical sign. For example, “ $\sqrt{4}$ ” is 2, because 2 times itself is 4. And “ $\sqrt{25}$ ” is 5, because 5 times itself is 25.



Q: When was math first mentioned in the Bible?

A: In Genesis, God told Adam and Eve to go forth and multiply.

✓ CHECK YOURSELF!

Which terms in this section were new to you? Reread the definition for any term that you did not already know.

Some Rules and Procedures

A few rules and procedures are fundamental to the study of statistics. Again, this review is intended for those whose math skills are rusty.

- *Multiplying by a reciprocal:* Dividing by a number is the same as multiplying by its reciprocal. For example, 6 divided by 3 is equal to 6 times $1/3$. Both are equal to 2. Some statistical formulas substitute one function for the other. If you are expecting division and don’t see it, check for multiplication by a reciprocal. For example, some formulas multiply by $1/N$ rather than divide by N .
- If the signs of two numbers being multiplied differ, the result will be negative. However, if the signs of the two numbers are the same (whether positive or negative), the result will be positive. You may have learned this rule as “a negative times a negative is a positive.” For example, $(+2)(+4) = +8$, and $(-2)(-4) = +8$. But $(+2)(-4) = -8$, because the signs of the numbers being multiplied differ.
- *Converting between fractions, decimals, and percentages:* To convert a fraction to a decimal, divide the numerator by the denominator. For example, $1/4$ is 1 divided by 4, which, when you carry out the division, is 0.25. To convert a decimal to a fraction, remove the decimal point and place the number over 1 followed by as many zeros as there were



Q: Where do mathematicians shop?

A: At the decimal.

original digits. For example, 0.25 is $25/100$, which further reduces to $1/4$. To convert a decimal to a percentage, multiply by 100 and add a percent sign. (To multiply by 100, move the decimal place two places to the right.) For example, 0.25 is 25%.

- *Deciding on the number of decimal places:* In math, “precision” refers to the exactness of a calculation. In the social sciences, two decimal places are usually considered adequate for a final answer. Answers carried to additional decimal places imply a degree of precision that we rarely have in the social sciences. However, most of the formulas in this textbook require that you perform a series of operations before arriving at a final answer. Rarely will either the final answer or the intervening steps be in whole numbers. If final answers are to be reported to two decimal places, you should carry all the preliminary steps to three decimal places and then round the final answer to two decimal places.

• A small error in the beginning is a great one in the end.

—St. Thomas Aquinas

Never round to whole numbers during the preliminary steps, as a series of such roundings may significantly alter the final answer.

- *Rounding numbers:* If the last digit is greater than 5, round up by adding one to the preceding digit. If the last digit is less than 5, leave the preceding digit unchanged. For example, 46.268 rounds up to 46.27, and 46.263 rounds down to 46.26. If the last digit is exactly 5, whether or not you round depends on the preceding digit. If the preceding digit is odd, round up by adding one to it; if the preceding digit is even, leave it unchanged. For example, 32.635 rounds to 32.64, but 32.645 also rounds to 32.64.

• Round numbers are always false.

—Samuel Johnson

- *Ordering operations:* When several arithmetic operations are called for in a formula, perform them in the following order: (1) squares or square roots (exponents), (2) multiplication or division, and (3) addition or subtraction. However, parentheses override that order. Operations within parentheses are always done first, regardless of what those operations are. If parentheses are stacked, do operations within the innermost parentheses first and work your way outward. You may have learned the order of operations as “Please Excuse My Dear Aunt Sally.” The first letters of that mnemonic stand for parentheses, exponents, multiplication, division, addition, and subtraction. For example, $4 + 2 * 3 = 4 + 6 = 10$. But $(4 + 2) * 3 = 6 * 3 = 18$. For another example, $2 + 3^2/2 = 2 + (3^2/2) = 2 + (9/2) = 2 + 4.5 = 6.5$. But $(2 + 3^2)/2 = (2 + 9)/2 = 11/2 = 5.5$.
- *Reordering terms to solve for an unknown:* If the quantity for which we want to solve is not alone on one side of an equation, we must first isolate it. For example, if we have the equation $24 = 16 + a$, we must isolate a on one side or the other of the equal sign. We do this by subtracting 16 from both sides. This gives us $24 - 16 = a$. This, of course, is 8.
 - Sometimes, a formula is given to solve for one term, but we instead want to solve for some other term within the formula. Again, we must reorder the terms to isolate the desired term. Take the formula for a z score, which you will come across in Module 8:

$$z = \frac{X - M}{s}$$

The formula solves for z . But what if we know z and want to instead solve for X ? We have to reorder the terms to isolate X . First, we get rid of the s in the denominator by multiplying both sides of the equation by s . This gives us $zs = X - M$. Then, we add M to both sides of the equation. This gives us $M + zs = X$. If we like, we can flip the terms on both sides of the equation so that X is to the left of the equal sign: $X = M + zs$. Either version is correct.

✓ CHECK YOURSELF!

Which rules and procedures in this section were new to you? Reread the explanation for any rule or procedure that you did not already know.

✓ CHECK YOURSELF!

Here are two formulas for a variance (a statistic that you will study later in the course):

$$s^2 = \sum (X - M)^2 (1/n) \quad \text{and} \quad s^2 = \frac{\sum (X - M)^2}{n}$$

Explain why both formulas lead to the same answer.



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PRACTICE

1. Complete the following chart:

<i>Fraction</i>	<i>Percentage</i>	<i>Decimal</i>
$\approx 33 \frac{1}{3}$	12.5%	0.667

2. Complete the following chart:

<i>Fraction</i>	<i>Percentage</i>	<i>Decimal</i>
$\frac{1}{6}$	10%	0.143

3. Complete the following chart:

<i>Fraction</i>	<i>Percentage</i>	<i>Decimal</i>
$\frac{1}{20}$	95%	0.25

4. Complete the following chart:

<i>Fraction</i>	<i>Percentage</i>	<i>Decimal</i>
$\frac{1}{50}$.99%	0.01

5. Round the following numbers to two decimal places:

- 26.412 _____
- 62.745 _____
- 36.846 _____

6. Round the following numbers to two decimal places:

- 77.935 _____
- 1086.267 _____
- 39.633 _____

7. Round the following numbers to two decimal places:

- 95.555 _____
- 00.023 _____
- 48.950 _____

8. Round the following numbers to two decimal places:

- 110.001 _____
- 12.635 _____
- 276.772 _____

9. Solve the following equations applying the rules of order of operations:

- $4 * 5 + 3 * 2 =$ _____
- $4(5 + 3) * 2 =$ _____
- $((4 * 5) + 3)(2) =$ _____
- $\sqrt{(4(5 + 3))}(2) =$ _____
- $4^2(5 + 3)(2) =$ _____

10. Solve the following equations applying the rules of order of operations:

- $4\sqrt{5^2}(3 * 2) =$ _____
- $(4)(5 + 3 * 2) =$ _____
- $4\sqrt{5} + (3 * 2) =$ _____
- $4^2(5 + 3 * 2) =$ _____
- $(4)(5 + 3)2^2 =$ _____

11. Solve the following equations applying the rules of order of operations:

- a. $4\sqrt{36} * (2 * 3) + 2 =$
- b. $6^2/12 - 4/2 =$
- c. $\sqrt{8 - 4} * 6/2 - 1 =$
- d. $[(30) (.5) - 6] * 2 + 4 =$
- e. $5 + 3 * 7 - 4 =$

12. Solve the following equations applying the rules of order of operations:

- a. $\sqrt{5 * 6 + 6 - 3^2} - 2 =$
- b. $(4 + 7^2 - 17)/6 =$
- c. $6 - 2 * 2 + 9/3 =$
- d. $100 - 10 * 5 - 5 =$
- e. $\sqrt{(12 - 8) * 16} - 2 =$

13. Reexpress the following equations, substituting reciprocals for division. Then, solve in decimal form.

<i>Equation</i>	<i>Reexpressed in Reciprocals</i>	<i>Solution</i>
a. $16/5 - 0.246 =$	_____ =	_____
b. $68 + 68/3 =$	_____ =	_____
c. $2/3 - 1/5 =$	_____ =	_____

14. Reexpress the following equations, substituting reciprocals for division. Then, solve in decimal form.

<i>Equation</i>	<i>Reexpressed in Reciprocals</i>	<i>Solution</i>
a. $98 - 75/3 =$	_____ =	_____
b. $672 + 2/6 =$	_____ =	_____
c. $8/3 + 5/4 =$	_____ =	_____

15. Reexpress the following equations, substituting reciprocals for division. Then, solve in decimal form.

<i>Equation</i>	<i>Reexpressed in Reciprocals</i>	<i>Solution</i>
a. $6/3 - 0.95 =$	_____ =	_____
b. $12.50 - 8/2 =$	_____ =	_____
c. $25/5 + 6 =$	_____ =	_____

16. Reexpress the following equations, substituting reciprocals for division. Then, solve in decimal form.

<i>Equation</i>	<i>Reexpressed in Reciprocals</i>	<i>Solution</i>
a. $155 + 5/2 =$	_____ =	_____
b. $3.28 - 3/2 =$	_____ =	_____
c. $720/9 - 45 =$	_____ =	_____

17. Rearrange the equation to solve for the indicated unknown.
- a. $64/4 + b = 30$ Solve for b : _____
- b. $0.30c = 20$ Solve for c : _____
- c. $Y = bX + a$ Solve for a : _____
18. Rearrange the equation to solve for the indicated unknown.
- a. $b + 20 = 0.5$ Solve for b : _____
- b. $F = 1.8c + 32$ Solve for c : _____
- c. $50 = 60/a$ Solve for a : _____
19. Rearrange the equation to solve for the indicated unknown.
- a. $b + 24/8 = 3$ Solve for b : _____
- b. $T = 12c - 4$ Solve for c : _____
- c. $12 = 3a$ Solve for a : _____
20. Rearrange the equation to solve for the indicated unknown.
- a. $7 = .5b + 2.5$ Solve for b : _____
- b. $64 = 4c$ Solve for c : _____
- c. $3a = .25V$ Solve for a : _____

More Rules and Procedures

In this textbook, derivations and proofs are kept to a minimum. However, a fuller understanding of the formulas and of the relationship between various statistics requires mathematical manipulations beyond the basic rules just presented. If your instructor intends to derive one formula from another (say, a raw score formula from a deviation score formula) or prove theorems (say, the binomial theorem), some additional mathematical rules are necessary. Knowing these rules is especially appropriate for students intending to take additional courses in statistics. Let your instructor be your guide on the need for this section.

- Rules for summation across and within parentheses
 1. The summation of a variable is the sum of that variable.

$$\Sigma(X) = \Sigma X$$

For example, if $X_1 = 2$, $X_2 = 4$, and $X_3 = 6$, then

$$\Sigma(X) = \Sigma X = 2 + 4 + 6 = 12$$

2. The summation of a constant is N times the constant.

$$\Sigma(a) = Na$$

For example, if $a = 6.24$ and there are 3 subjects, then

$$\Sigma(a) = Na = 3 * 6.24 = 18.72$$

3. The summation of a constant times a variable is the constant times the sum of the variable.

$$\Sigma(aX) = a\Sigma X$$

For example, if $a = 6.24$ and if $X_1 = 2$, $X_2 = 4$, and $X_3 = 6$, then

$$\Sigma(aX) = a\Sigma X = 6.24 * (2 + 4 + 6) = 6.24 * 12 = 74.88$$



Do not worry about your difficulties in mathematics. I assure you that mine are greater.

—Albert Einstein

4. The summation of two or more terms within parentheses is the same as their independent summation.

$$\Sigma(X + Y) = \Sigma X + \Sigma Y$$

$$\Sigma(X + a) = \Sigma X + Na$$

$$\Sigma(X + aY) = \Sigma X + a\Sigma Y$$

For example, if $X_1 = 2$, $X_2 = 4$, and $X_3 = 6$ and if $Y_1 = 1$, $Y_2 = 3$, and $Y_3 = 5$ and if $a = 6.24$, then

$$\Sigma(X + Y) = \Sigma X + \Sigma Y = (2 + 4 + 6) + (1 + 3 + 5) = 12 + 9 = 21$$

$$\Sigma(X + a) = \Sigma X + Na = (2 + 4 + 6) + (3 * 6.24) = 12 + 18.72 = 30.72$$

$$\Sigma(X + aY) = \Sigma X + a\Sigma Y = (2 + 4 + 6) + (6.24)(1 + 3 + 5) = 12 + (6.24)(9) = 12 + 56.16 = 68.16$$

- Rules for expanding binomials

1. $(ab)^2 = (ab)(ab) = aa * bb = a^2 * b^2$

Thus, $(ab)^2 = a^2 * b^2$

For example, $(4 * 3)^2 = 4^2 * 3^2 = 16 * 9 = 144$.

2. $(a + b)^2 = aa + ab + ab + bb = a^2 + 2ab + b^2 = a^2 + b^2 + 2ab$

Thus, $(a + b)^2 = a^2 + b^2 + 2ab$

For example, $(4 + 3)^2 = 4^2 + 3^2 + 2(4 * 3) = 16 + 9 + 2(12) = 25 + 24 = 49$

3. $(a - b)^2 = aa - ab - ab + bb = a^2 - 2ab + b^2 = a^2 + b^2 - 2ab$

Thus, $(a - b)^2 = a^2 + b^2 - 2ab$

For example, $(4 - 3)^2 = 4^2 + 3^2 - 2(4 * 3) = 16 + 9 - 2(12) = 25 - 24 = 1$

PRACTICE

21. Expand the following expressions using the rules for summation within parentheses and for binomial expansion (*Note:* Numerals are always constants; thus, N is always a constant):

a. $\Sigma(X + Y)^2$ _____

b. $\Sigma(bXY + 1)^2$ _____

c. $\Sigma(XY + 1)$ _____

22. Expand the following expressions, using the rules for summation within parentheses and for binomial expansion (*Note:* Numerals are always constants; thus, N is always a constant):

a. $\Sigma(1 + N)$ _____

b. $\Sigma(2X + Y)$ _____

c. $\Sigma(aX)^2$ _____

That should be all the math you need in this textbook. Of course, higher math is necessary for the fullest understanding of statistical formulas and for further study in statistics. However, a first course in statistics rarely goes beyond what is presented here.



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