Introduction

What Is Teaching by Design?

Teaching by Design in Elementary Mathematics is a series of professional development guides that helps teachers create and share knowledge for teaching mathematics. This guide is one of three volumes in a set that focuses on topics within number and operations from kindergarten through Grade 5. Each volume in the series is organized in 14 professional development sessions in which teams of teachers learn, share, and plan together in a structured, collaborative environment.

Through participation in the carefully sequenced sessions in each guide, teachers build the specialized understanding of mathematics and pedagogy that supports effective instruction. The culmination of this professional learning process is the development of a prototype lesson, a mathematics lesson collaboratively designed by the team. In teaching this prototype lesson in one or more teachers’ classrooms, the team can investigate its impact on student learning. The cycle of investigating, planning, teaching, observing, debriefing, and revising a lesson together contributes to a climate of continuous professional learning.

Expected Outcomes for Teaching by Design in Elementary Mathematics

1. Teachers will deepen their content knowledge of important mathematical concepts for the grade level they teach.
2. Teachers will increase their understanding of how students learn these mathematical ideas.
3. Teachers will use their knowledge to develop effective lessons and improve instruction.
4. Teachers will enhance their collaboration skills.

Mathematical Content Topics

Several sources were consulted to identify the mathematical focus for each volume in the Teaching by Design series. The content is aligned with the topics in number and operations identified by the National Council of Teachers of Mathematics (NCTM) in Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics (2006). The focal points help teachers to hone in on the most important topics, make connections between topics, and to provide students with an integrated understanding of mathematics. The mathematical content of Teaching by Design also matches content topics in number and operations found in the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Number and operations was selected as the content area for these materials because it is the cornerstone of mathematics education in Grades K–5. All mathematics is grounded in number, including algebra, geometry, measurement, and statistics (NCTM, 2000). This area
of the curriculum is often considered to be the most simple and straightforward because it is the mathematics with which young children start their education. But learning about number and operations is in fact a complex process that has been a primary area of research in mathematics education (Fuson, 2003; Kilpatrick, Swafford, & Findell, 2001).

**Model of Professional Development**

One of the strengths of the *Teaching by Design in Elementary Mathematics* process is its alignment with the characteristics of effective professional development for teachers (Borasi & Fonzi, 2002; Corcoran, 1995; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hawley & Valli, 1999; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Wilson & Berne, 1999). Specifically, effective learning experiences for teachers include the following characteristics:

- Opportunities for collaboration
- Ongoing activities
- Focus on content
- Teacher-driven and classroom-based methods
- Active and hands-on activities
- Focus on student learning

The *Teaching by Design* approach has been developed with these characteristics in mind to preserve this connection to high-quality professional development.

Teachers who are implementing any model for collaborative professional development can use *Teaching by Design*. Figure 1 provides a list describing some of these models and their features. Teacher teams that have already established one of these models for ongoing, job-embedded professional development will find that *Teaching by Design* is easily implemented within any of these models.

Teachers familiar with lesson study will recognize many of its features in the *Teaching by Design* materials (Fernandez & Chokshi, 2002; Lewis, 2002; Stigler & Hiebert, 1999; Yoshida, 1999). The sessions in each volume have been structured to provide support for the powerful practices of lesson study within any collaborative professional development model. Some of the practices from lesson study that will be evident in the *Teaching by Design* volumes include the following activities:

- Analyzing teaching materials
- Focusing on important content
- Establishing precise and connected lesson goals
- Developing well-planned lessons
- Observing student learning
- Analyzing student understanding

Teacher teams that have not previously engaged in one of these professional development models will still be able to successfully implement *Teaching by Design*. An informal group of grade-level teaching partners can use the materials provided as a way to take a first step toward establishing one of these professional development models. The resources section includes some recommended readings about each model that will assist groups that want to know more about them.
**Collaborating to Design a Prototype Lesson**

**Why Design?**

The term *design* generally refers to the creation of a product in an artistic or highly skilled manner, and it is usually associated with the applied arts, engineering, and architecture. In these contexts, good design means that the product fits the needs of the people who will use it and the context in which it will be used.

One of the more specific definitions of design is the process of preparing a detailed and deliberate plan for accomplishing something (Merriam-Webster, 1993). This definition is the essence of *Teaching by Design*. It is a way to describe teachers’ work that focuses on planning well-designed lessons that fit the needs of their students.

Some of the general practices of design have parallels with teaching, and include the following practices:

- Identifying and framing problems and needs
- Working collaboratively

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**Figure 1  Models of Professional Development**

<table>
<thead>
<tr>
<th>Critical friends groups (CFG)</th>
<th>A group of teachers work together to examine and change their classroom practice guided by established group processes and protocols.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical features:</td>
<td>A CFG coach guides the teachers’ work.</td>
</tr>
<tr>
<td></td>
<td>The team uses CFG protocols to structure conversations about curriculum, student work, or relevant readings.</td>
</tr>
<tr>
<td>Lesson study</td>
<td>Teachers collaborate to develop a lesson plan, teach and observe the lesson to collect data on student learning, and use their observations to refine their lesson.</td>
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<tr>
<td>Typical features:</td>
<td>Lesson study teams are self-directed and democratic, but may work with a facilitator and outside advisors.</td>
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<td></td>
<td>The team has established protocols for designing, observing, debriefing, and revising collaboratively designed lessons.</td>
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<tr>
<td>Mathematics coaching</td>
<td>A specialist works with teachers one-on-one to examine and improve classroom practice and to improve pedagogical content knowledge.</td>
</tr>
<tr>
<td>Typical features:</td>
<td>The mathematics coach works with individual teachers or grade-level teams of teachers.</td>
</tr>
<tr>
<td></td>
<td>The coach and teachers commonly observe each other teaching a lesson or coteach lessons.</td>
</tr>
<tr>
<td>Professional learning teams</td>
<td>Small groups of teachers work together to improve instruction and student learning.</td>
</tr>
<tr>
<td>Typical features:</td>
<td>Members of the group share leadership of the group.</td>
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<td></td>
<td>The team engages in reading and discussion on topics of professional interest to the group.</td>
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</tbody>
</table>
• Gathering and analyzing information
• Determining performance criteria for successful solutions
• Generating alternative solutions and building prototypes
• Evaluating and selecting appropriate solutions
• Implementing choices
• Evaluating outcomes

"A teacher ideally conceived is a designer who helps learners design themselves.”
—David Perkins, *Knowledge as Design*

When architects or mechanical designers work on a project, they almost always collaborate with other professionals. Designing a building, a computer, or a refrigerator is complicated work and requires the best thinking of a team of experts. Similarly, well-designed lessons are often the product of several teachers working together to think deeply about the goals and strategize the best ways to help students achieve them.

In Sessions 2 through 11 of *Teaching by Design*, you will discuss and explore aspects of mathematics teaching and learning that will prepare you for Session 12, in which you will work together as a design team to develop a well-planned lesson. After the lesson has been taught in one or more classes, in Sessions 13 and 14 you will discuss the lesson results, then revise and improve the lesson. While the focus is on a single lesson designed by the group, the learning gained from the collaborative process will influence the many other lessons you plan individually.

**Why Prototype Lessons?**

In product design, a prototype is the original form of the product that serves as the basis or standard for other versions of that product. Through the *Teaching by Design* process, teachers will collaborate to design a prototype lesson. This lesson is the context through which teachers explore their ideas and questions about how students learn challenging mathematical concepts. As they design the prototype lesson together, teachers explore how to sequence learning experiences that engage students with important mathematics while strengthening students’ problem-solving abilities. For designers, the implementation of a prototype provides information to enhance future products. Likewise, the implementation of the prototype lesson provides teaching teams with insights about teaching and learning that can be applied to the development and delivery of future lessons.

The prototype lesson provides an opportunity to incorporate the mathematical and pedagogical knowledge teachers have gained through the *Teaching by Design* sequence. The resulting lesson plan is more detailed than a typical lesson plan and represents teachers’ collective ideas about helping students understand important mathematical ideas. In addition to laying out the learning activities and the sequence of instruction, the plan for the prototype lesson includes background information about the instructional decisions that have been made, goals for student learning, carefully worded questions and prompts, anticipated student responses and teacher supports, and points of evaluation. Including all these elements in the lesson plan facilitates the observation, debriefing, and revision of the lesson. It also serves as a record of the professional learning for this team while they are engaged in these professional development sessions.
The prototype lesson offers teachers a perfect opportunity to apply their knowledge and understanding of the following areas:

- **Mathematics content**, including their understanding of how the mathematical concepts are interconnected
- **Students’ prior knowledge**, including what students have learned in previous lessons and what they have learned in prior grade levels or through experiences outside the classroom
- **Learning progressions**, including ideas of how students develop increasingly sophisticated strategies, big ideas, and models

### Overview of the Sessions

The materials are organized into a series of 14 work sessions, each approximately 90 minutes long.

- Session 1  Getting Started
- Session 2  Learning Landscape
- Session 3  Addition and Subtraction With Like and Related Fractions
- Session 4  Understanding Equivalent Fractions
- Session 5  Problem Solving With Rational Numbers
- Session 6  Decimal and Fraction Relationships
- Session 7  Comparing and Ordering Decimals and Fractions
- Session 8  Mathematical Models: Adding and Subtracting Fractions
- Session 9  Mathematical Games: Adding and Subtracting Fractions and Decimals
- Session 10 Understanding Standard Procedures
- Session 11 Instruction to Support Fluency With Standard Procedures
- Session 12 Designing the Prototype Lesson
- Session 13 Discussing Results
- Session 14 Reflecting On and Revising the Prototype Lesson

Sessions 1 and 2 establish a context for the work. In Session 1, teams will establish group norms and explore design principles. In Session 2, teachers will begin to develop components of a learning landscape for number and operations in fourth and fifth grades.

Sessions 3 through 11 focus on how children learn specific mathematical topics. The activities include opportunities to do mathematics problems, to examine how students learn, and to analyze and discuss student work. Some of the sessions include activities that focus on how the ideas from that session can be used to inform instruction.

Between sessions, teachers will engage in two types of activities. Student Connections activities are opportunities for teachers to observe student learning and collect student work. Investigating Instructional Materials activities are opportunities to analyze teaching resources. These activities are integral to subsequent sessions because they are used to help illustrate important mathematical and pedagogical ideas and allow teachers to apply their learning to their practice.
During Session 11, the team begins work on the prototype lesson. The process of designing the lesson may take more than one meeting to complete. When the lesson is ready, one or more team members implement the lesson with their students. Session 13 should occur after the teaching of the lesson and is structured to provide teachers with an opportunity to discuss what happened and to analyze evidence of student learning. Depending on how many teachers implement the lesson, some teams will need additional time for this discussion. Session 14 gives teachers the opportunity to revise the lesson, reflect on the professional development process, and identify next steps.

Many times throughout Teaching by Design, you will be directed to write in your journal. It will be helpful to organize your journal into three sections: (1) Activities, (2) Student Connections, and (3) Lesson Design Notes. We suggest using either a binder with tabs or a composition book or spiral notebook with tabbed sections in which you take notes and staple handouts.

**Facilitating**

Each of the sessions is designed to be facilitated by one person. The facilitator can be a coach or team leader, or the facilitation role can rotate within the group. When team members take turns serving in this role, all of the teachers receive an opportunity to develop their leadership skills. In either case, the facilitator should take responsibility for previewing the session content and making sure that all the materials are prepared. During the session, the facilitator should also serve as the timekeeper and maintain the pace of the session.

Each session includes Facilitator Notes, outlining the specific responsibilities for that session. The notes include additional information the facilitator can use to support the work of the team. One of the key responsibilities during facilitation is helping to maintain the group norms that the team identifies in the first session. It is usually helpful for the team to revisit the list of group norms periodically to make sure they are adhering to the norms and to identify any additions or modifications to the list.

Another key responsibility for the facilitator is listening actively to the group’s conversations throughout the session. The facilitator will help monitor the discussions and activities to ensure that all team members have an opportunity to participate, share their knowledge, and learn. The facilitator can also help get the discussions started with an initial observation or question if there is a lag in the conversation.

If a leader or coach will serve as the facilitator, keep in mind that the team is intended to be self-directed. All members of the team are responsible for making decisions, contributing equally to the work, and supporting their colleagues’ professional learning. The resources listed at the end of this section provide for more information on the skills and responsibilities of facilitators in different types of professional learning teams.

**Finding Time for Collaboration**

Teaching by Design in Elementary Mathematics is intended to guide teams of teachers through a process of collaborative investigation that will deepen their knowledge for teaching mathematics. To engage in Teaching by Design sessions, teachers will need time to meet together. Ideally, each group will meet regularly and frequently. Having weekly or biweekly meetings keeps the momentum going while also allowing time for collecting student data between the sessions.
When opportunities for teacher collaboration are a regular part of the school schedule, finding time should not be an issue. However, even if collaboration time is not part of the existing schedule, teams can find other ways to meet. For example, team meetings can be held outside regular school hours before or after school. Teams may find that they have occasional opportunities to work for extended sessions on early-release days or on days scheduled for professional development. If this is the case, groups can complete more than one 90-minute session in that setting and collect student data to share at a subsequent meeting.

There are a number of excellent resources that specifically address the issues associated with finding time within the school schedule for school-based professional development and teacher collaboration time. Figure 2 illustrates the strategies schools have used to create more time for teachers to work together.

**Figure 2  Strategies for Creating Time for Collaboration**

<table>
<thead>
<tr>
<th>Adjusting the School Schedule</th>
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<tbody>
<tr>
<td><strong>Early Release/Late Start</strong></td>
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<tr>
<td>Four days a week, the school schedule is extended by several minutes. One day a week, students come to school one hour late or go home one hour early. A variation on this schedule is to have teachers come to school 30 minutes early, with students arriving 30 minutes late; a similar strategy can also be used at the end of the day. (An early release/late start schedule does not have to be used on a weekly basis but can be spread out over the course of several weeks.)</td>
</tr>
<tr>
<td><strong>Professional Development Days</strong></td>
</tr>
<tr>
<td>The daily school schedule is extended by several minutes in order to release students for a full day once a month or every six weeks.</td>
</tr>
<tr>
<td><strong>Prep Time</strong></td>
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<tr>
<td>Teachers in each grade level or each department have a common prep time that can be used weekly (or as needed) to work together.</td>
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<tr>
<th>Covering Classes</th>
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<tbody>
<tr>
<td><strong>Specialist Days</strong></td>
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<tr>
<td>Each day of the week, students from one grade level spend most of the day with specialists, in the computer lab, and in the library.</td>
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<tr>
<td><strong>Service Learning</strong></td>
</tr>
<tr>
<td>For one half-day each week, students spend their time conducting service learning or community projects.</td>
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<tr>
<td><strong>Paraprofessionals/Administrators/Parents/Volunteers</strong></td>
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<tr>
<td>Teachers’ classes are taken over for one hour each week.</td>
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<tr>
<td><strong>Teaming</strong></td>
</tr>
<tr>
<td>Teacher teams pair up and take each other’s classes for one hour each week. For example, each second-grade teacher takes one class of first graders so that the first-grade teachers can meet together.</td>
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<tr>
<td><strong>Substitute Teachers</strong></td>
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<tr>
<td>Substitutes are hired to rotate through the classes one day every other week.</td>
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<tr>
<th>Reallocating Existing Time</th>
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<tbody>
<tr>
<td><strong>Staff Meetings</strong></td>
</tr>
<tr>
<td>Weekly staff meetings are cut back to once or twice a month and replaced with grade-level or department meetings.</td>
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<tr>
<td><strong>Adjusting Planning Time</strong></td>
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<tr>
<td>Teachers’ daily planning time is used for collaborative work one day a week. The number of minutes that teachers are expected to stay after school can be cut back by 10 minutes on four days during the week in order to create an extra 40 minutes for meeting together on one afternoon.</td>
</tr>
<tr>
<td><strong>Professional Development/Inservice Days</strong></td>
</tr>
<tr>
<td>Teachers are excused from staff development days to compensate for weekly meetings outside school hours.</td>
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</tbody>
</table>
References


Additional Resources

**Critical Friends Groups**
National School Reform Faculty
  http://www.nsrfrharmony.org/faq.html#1

Coalition of Essential Schools Northwest
  http://www.cesnorthwest.org/cfg.php


**Lesson Study**
Lesson Study Group at Mills College
  http://www.lessonresearch.net/

Lesson Study Northwest Regional Educational Laboratory. Center for Classroom Teaching and Learning
  http://educationnorthwest.org/service/235


**Mathematics Coaches**
Examining the Role of a Math Content Coach Eye on Education

Pedagogical Content Coaching Silicon Valley Mathematics Initiative


**Professional Learning Teams**


Professional Learning Teams (PLTs) to Improve Student Achievement Education Northwest
  http://educationnorthwest.org/service/295